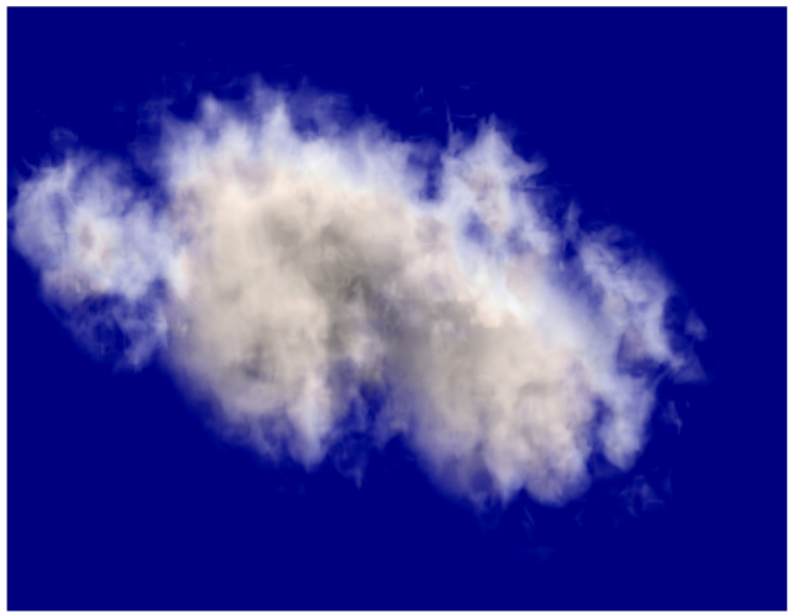
Maths 190 Lecture 16

- **Assignment 3:** due Wed May 12.
- **Lecture Wed May 12:** Test solutions and assignment questions.
- **Topic for today**: Fractal Dimension

- **Question of the day**: What is the dimension of a cloud?



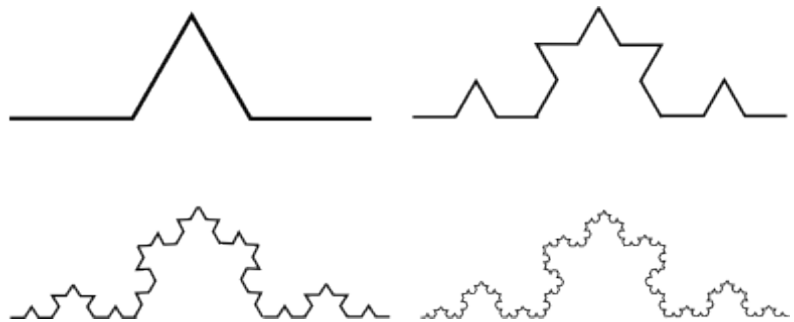
What do we mean by dimension?

Why do we say a line is one-dimensional but a square is two-dimensional?

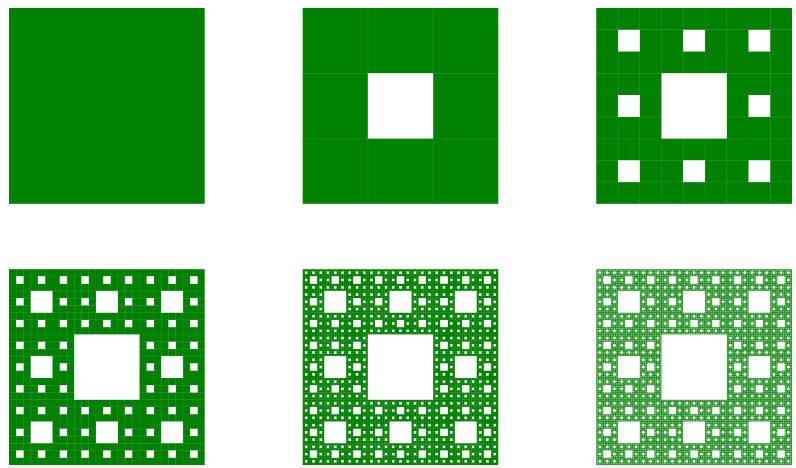
Why is a cube three-dimensional?

Fractal dimensions

- What is the dimension of the Koch curve?
  - It's not a straight line.
  - It's infinitely long and infinitely detailed



- What about the Seipinski Carpet?
  - It's not a solid and it has zero area.



- How can we calculation the dimension of these fractals?

A scaling problem

Start with a line segment of a fixed length.

- How many copies of the line segment does it take to make another line segment that is twice as long?
- How many copies of the line segment does it take to make another line segment that is three times as long?

Original object	Dimension of object	Scaling factor to make larger version	Number of copies to build larger version
Line	1	2	$2 = 2^1$
	1	3	
Square	2	2	
	2	3	
Cube	3	2	
	3	3	

A definition of dimension

Let d be the dimension of an object, let S be the scaling factor to make a larger version, and let N be the number of copies required to build the larger version.

We found the following pattern for line segments, squares and cubes..

$$S^d = N$$

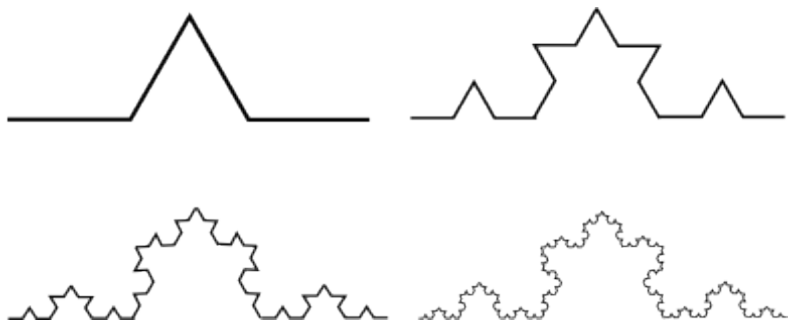
Use this to define dimension.

$$d = \frac{\ln N}{\ln S}$$

Here $\ln x$ means the "natural logarithm" of the number x.

Note: d does not need to be an integer for this definition to make sense.

Dimension of the Koch Curve



- What is the scaling factor?

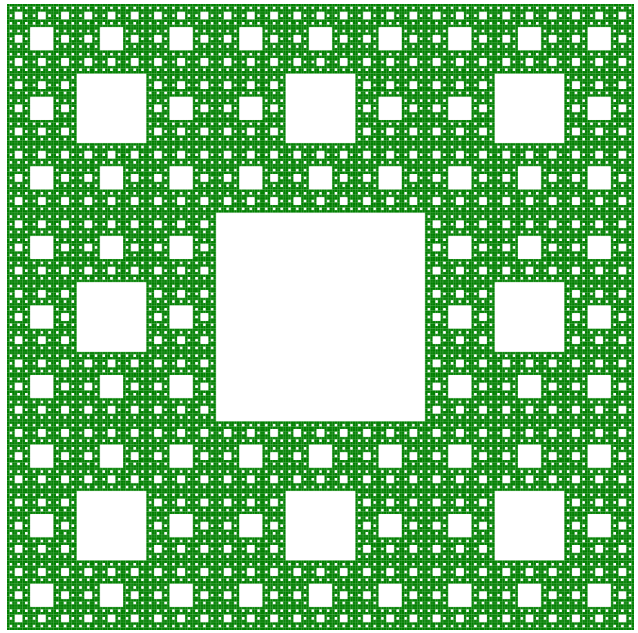
$$S =$$

- How many copies are needed?

$$N =$$

- What is the dimension?

Dimension of the Seirpinski Carpet



- What is the scaling factor?

$$S =$$

- How many copies are needed?

$$N =$$

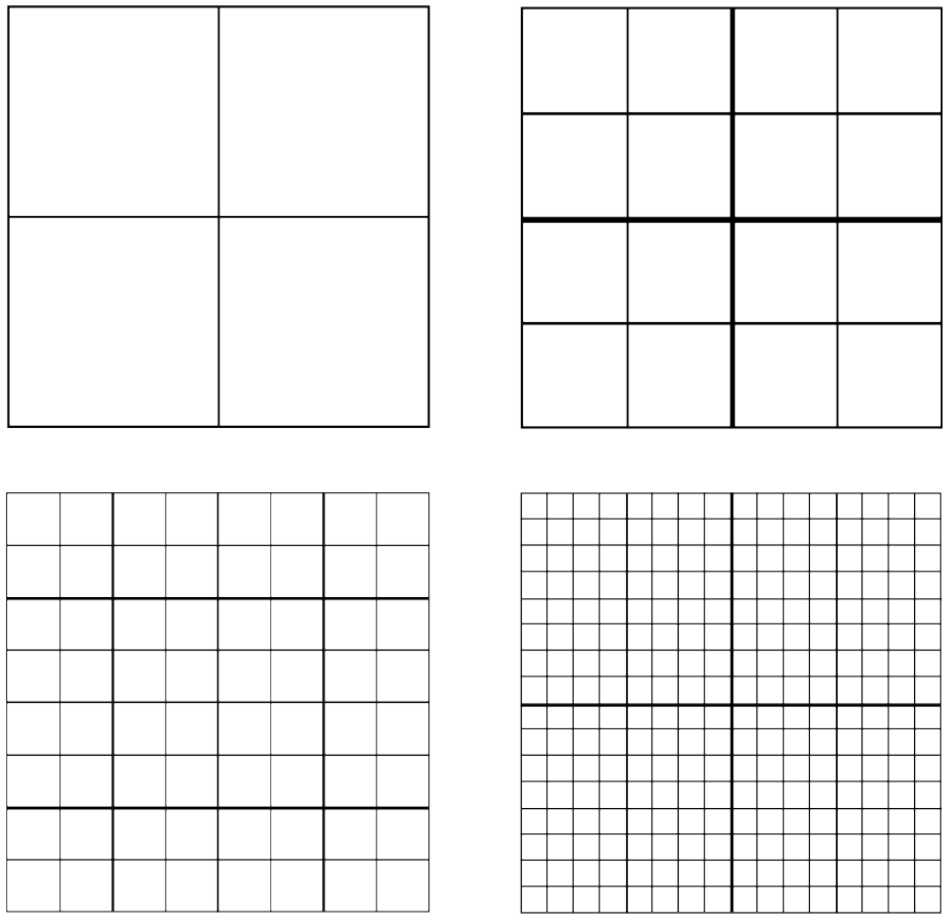
- What is the dimension?

Building fractals

- Can we create a fractal with dimension $\frac{\ln 3}{\ln 2}$?

Building fractals

- Can we create a fractal with dimension $\frac{\ln 3}{\ln 2}$?



Building fractals

- Can we create a fractal with dimension $\frac{\ln 3}{\ln 2}$ using line segments?

Important idea from today:

The dimension of an object need not be an integer.

We defined the notation of a “fractal dimension”.

For next time

- Read section 6.2
- Can you construct a curve with dimension exactly 1.5?