Maths 190 Lecture 14

REMINDER: Test tomorrow at 6:15. Room MLT1.

Topic for today: Tiling the plane

Question of the day: What is the most symmetric shape that can be used to tile the plane?

What is the least symmetric shape that can be used to tile the plane?

Tilings of the plane

In pairs or threes: Use the handout and the transparencies provided to investigate the symmetries corresponding to each of the tiling patterns on the front page of the handout. Write a list of the symmetries for each case.

Symmetry in tilings of the plane

We distinguish between two types of symmetries:

- A **rigid symmetry** of a pattern in the plane is a motion of the plane that preserves the pattern and does not shrink, stretch, or otherwise distort the pattern.

- A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.

Note that these definitions refer to symmetries of a pattern, not to symmetries of the individual tiles.

Rigid symmetry in the plane

All rigid symmetries are formed by combining the following three basic rigid symmetries:

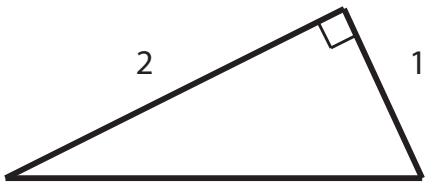
- shifts (also called translations)
- rotations around a point
- flips (also called reflections) along a line.

Theorem: Any rigid symmetry can be formed by first reflecting (if necessary), then rotating (if necessary), and finally translating (if necessary).

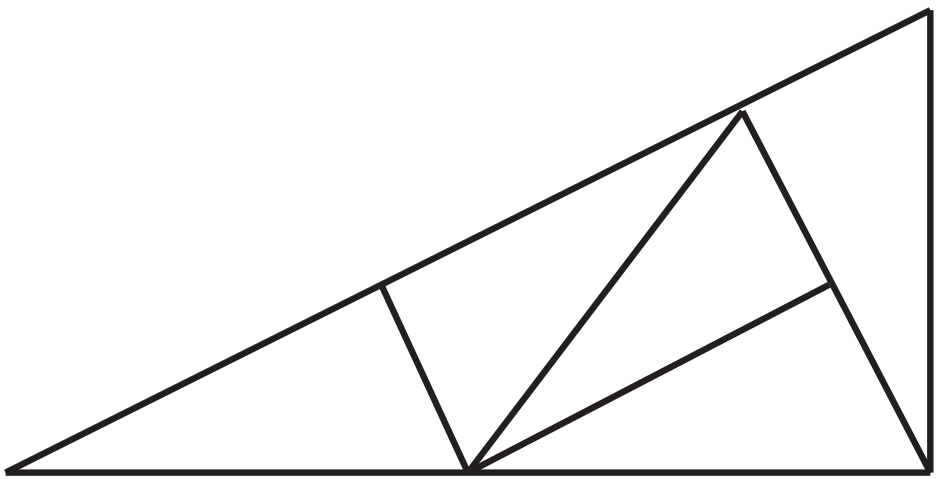
- Some tilings have rigid symmetries and symmetry of scale (e.g., tilings using square tiles or equilateral triangle tiles).
- Other tilings have rigid symmetries but no symmetry of scale (e.g., tilings using hexagonal tiles)
- Do any tilings have symmetry of scale but no rigid symmetries?

The Pinwheel Pattern

The pinwheel pattern is a tiling of the plane using a triangular tile: a right angle triangle with sides of length 1, 2 and $\sqrt{5}$ units.

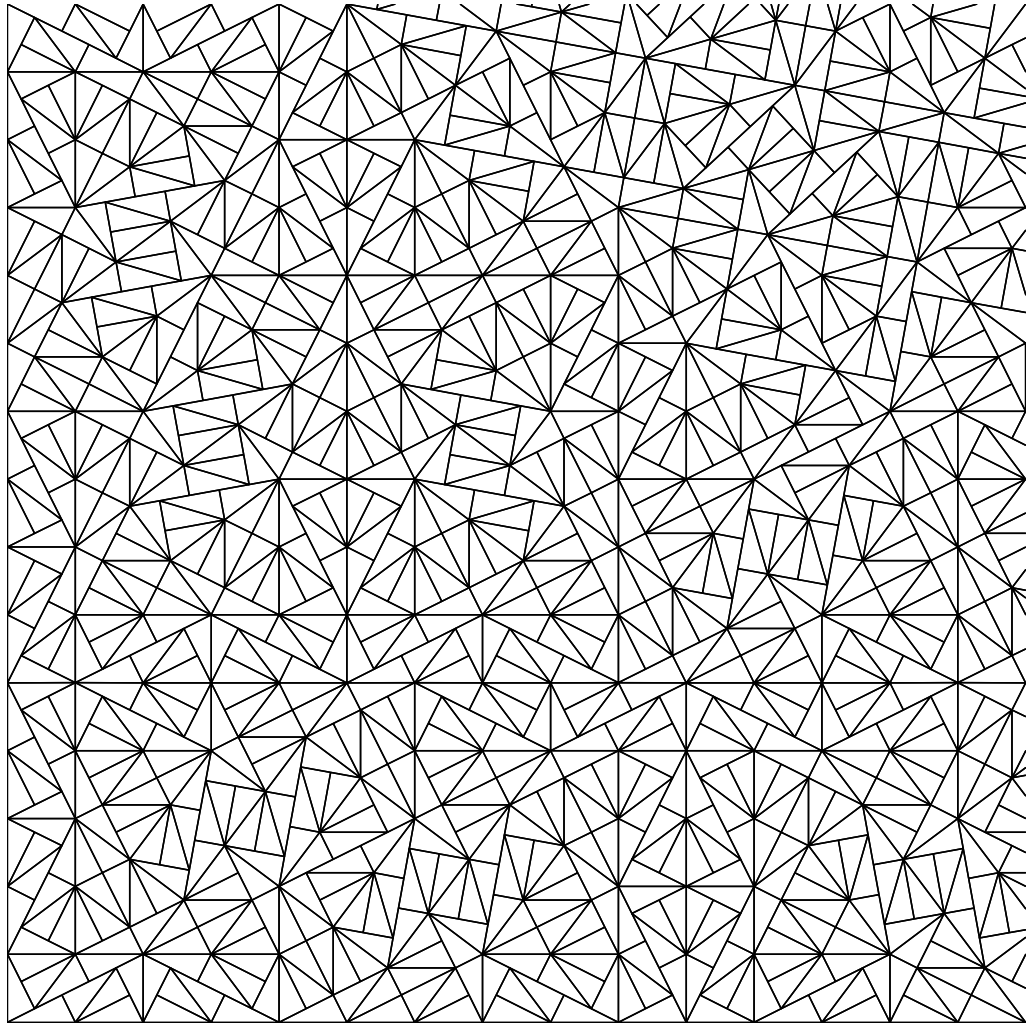


Five pinwheel triangles are combined to make one super-tile.



Five pinwheel super-tiles can be combined to make one super-super-tile, and so on . . .

The pinwheel pattern has symmetry of scale but no rigid symmetries. We say the pattern is **aperiodic**.



Important ideas from today:

- Patterns in the plane can have two types of symmetries: rigid symmetries and symmetry of scale.
- Some patterns have symmetry of scale but no rigid symmetries. The pinwheel pattern is an example of this type of pattern.

For next time

- Read §4.4 in the textbook.
- Try some Mindscapes at the end of §4.4 of textbook.
- Look about you as you go about your life in the next few days and notice the tiling patterns you see. What symmetries can you see?