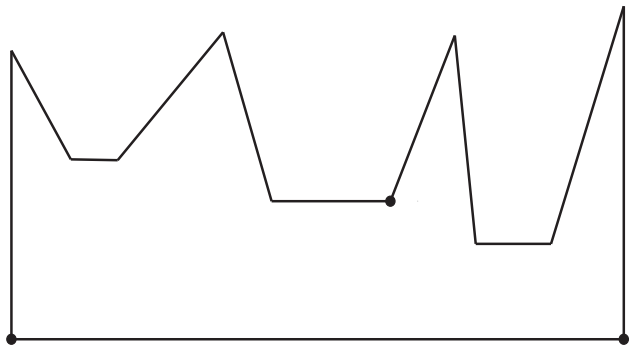
Department of Mathematics

Maths 190 Assignment 3 Solutions

October 12, 2009

1. (a) (3 marks) Three guards are needed. Two guards is not enough because it is not possible for two guards to between them see into all the "peaks" of the gallery. One possible placement of the guards is shown by the dots in the figure below.



   (b) (2 marks) The gallery has 12 sides. The Art Gallery Theorem predicts that no more than 12/3=4 guards is necessary, but makes no prediction about the minimum number of guards. Thus the answer in (a) is consistent with the Art Gallery Theorem.

   (c) (2 marks) It is not possible to draw a gallery with 10 straight sides that needs at least four guards to view it. The Art Gallery Theorem tells us that no more than n guards are needed, where n is the biggest integer that is less than or equal to 10/3=3.33... Thus a maximum of three guards is needed for any gallery with 10 straight sides.

2. (a) (3 marks) A **rigid symmetry** of a pattern in the plane is a motion of the plane that preserves the pattern and does not shrink, stretch, or otherwise distort the pattern. Examples of rigid symmetries are shifts, rotations and flips (or reflections). A pattern in the plane has a **symmetry of scale** if the tiles that make up the pattern can be grouped into super-tiles that still cover the plane and, if scaled down, can be rigidly moved to coincide with the original pattern.

   (b) (3 marks) There are lots of examples possible here.

   (c) (1 mark) Answer depends on the pattern.

3. (6 marks)

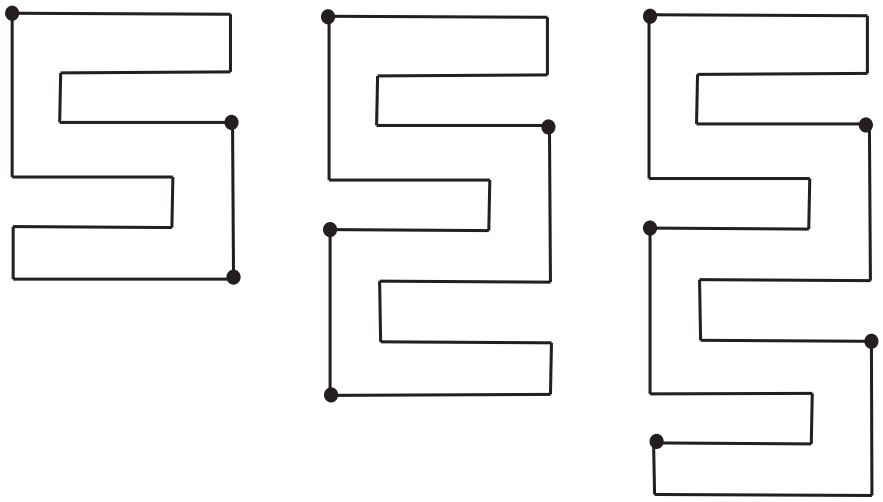
   (a) Five copies of the large square scaled down by a factor of three are needed to construct the second step in the collage process (and similarly, five copies of the second step scaled by a factor of three are need to construct the third step). Thus the fractal dimension is $\ln(5)/\ln(3) = 1.465$.

   (b) Six copies of the triangle scaled down by a factor of four are needed to take a step in the collage process so the fractal dimension is $\ln(6)/\ln(4) = 1.2925$.

   (c) Four copies of the line segment scaled down by a factor of three are needed to take a step in the collage process so the fractal dimension is $\ln(4)/\ln(3) = 1.2619$.

Tutorial questions:

Tutorial 6: (6 marks, 2 for each correct case) There are lots of possible answers. For instance, see the shapes below.



Tutorial 7: (5 marks) For each solid, compute the number of faces that meet at a vertex. Then the shape of the boundary of the new cut has as many sides as this. So, for the tetrahedron, cube and dodecahedron, the boundary of the cut is a triangle. For the octagon, the boundary of the cut is a square. For the icosahedron, the boundary of the cut is a pentagon.

Tutorial 8: (4 marks) There are lots of possible answers. The important point is that in going from the zeroth to first step (or first to second step), 5 copies of the original shape should be used, each scaled down by a factor of 4.