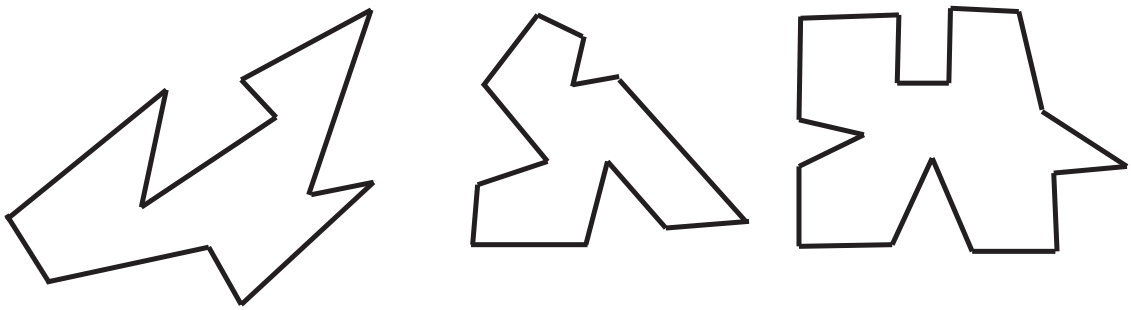
DEPARTMENT OF MATHEMATICS

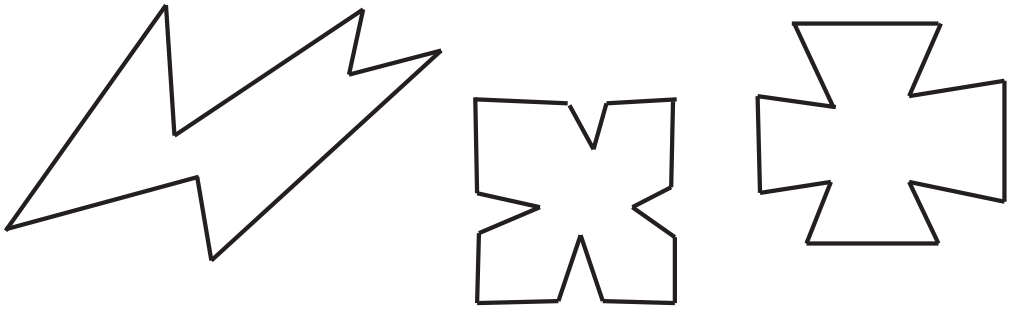
MATHS 190 Tutorial 6

Discuss the following situations with one or two other students and try to work out together a solution to each question. Make sure everyone in the group contributes to the discussion. When you have an answer to a question that everyone agrees with, make some notes outlining how you worked out the answer. Then try to explain your answer to your tutor.

1. (Mindscape 7, §4.2 of text) For each of the following floor plans, place guards at appropriate vertices so that every point in the museum is within view of at least one guard.



2. (Mindscape 9, §4.2 of text) Divide each of the following floor plans into triangles by adding straight segments that do not cross each other yet span the inside of the closed curve and extend from one vertex to another.



3. For each of the triangulations you found in question 2, colour the vertices of the floor plan red, blue or yellow so that every triangle has all three colours. What can you say about the number of guards needed to guard each of these galleries?

4. Draw the floor plan of a gallery with eleven sides which needs exactly one guard so that the entire gallery can be viewed. Draw another gallery with eleven sides that needs two guards. Draw another gallery with eleven sides that needs three guards. Show the placement of guards in your drawings.

   Is it possible to draw a gallery with eleven sides that needs at least four guards? If so, draw the gallery and show the placement of guards. If not, explain why not.

5. **Write up your answer to this question and hand it in with your answers to Assignment 3 (due September 30th).** Don't forget to write down the names of the people in your tutorial group, so that you can acknowledge your collaborators in your report.

   (Mindscape 20, §4.2 of text) Draw examples of museums with only right-angled corners having 12 sides, 16 sides and 20 sides, that require three, four, and five guards, respectively. Show the placement of guards in your drawings.

6. Draw shapes with three, four, six and eight sides that can be used to tile the plane. Find at least two different tiles for the case of six sides. For each shape you find, draw a tiling pattern.

7. For each tiling pattern you found in the previous question, find all the rigid symmetries and symmetries of scale.

8. (Mindscape 12, §4.4 of text) The picture on the next page shows a Pinwheel Pattern on one side. For each of the shaded tiles, outline the surrounding tiles that create the 5-unit super-tile, the 25-unit super-super-tile and the 125 unit super-super-super-tile. There is only one correct answer.

9. (Mindscape 15, §4.4 of text) On the other side of the attached sheet are pictures of the square and equilateral triangle patterns. For each of the shaded tiles, outline surrounding tiles that create the 4-unit super-tile and the 16-unit super-super-tile. In each case, show that there is more than one correct answer.

10. **Challenge question:** Draw a gallery with 11 sides such that

    (a) there is no point in the gallery from which all points in the gallery are visible directly, but

    (b) if all the internal walls are mirrored, there is a point inside the gallery from which the whole gallery is visible either directly or using the mirrors.

    In general, if all the internal walls of a gallery are mirrored, is it always possible to find a point in the gallery from which all of the gallery is visible?