

- ▶ Topic for today: **The irrational side of numbers**
- ▶ **Vitally important question of the day:**

How many rational numbers are there?

What is a rational number?

- ▶ A rational number x is one that can be written in the form

$$x = \frac{n}{m}$$

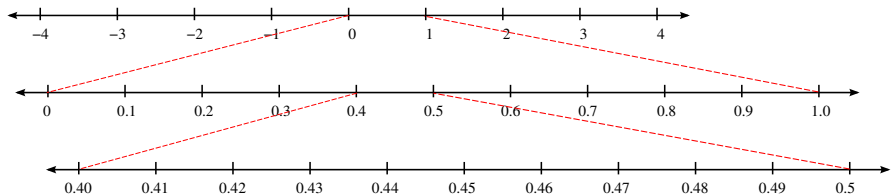
for some integers n and m .

- ▶ If we can write a number as a terminating decimal, then it is rational

$$0.675 = \frac{675}{1000} = \dots$$

- ▶ Rational numbers written as fractions can be reduced so that the top and bottom have no common factors.

How many rational numbers are there between 0 and 1?



- ▶ We can always find a rational number between two other rational numbers.
- ▶ If a and b are rational, then

$$\frac{a + b}{2}$$

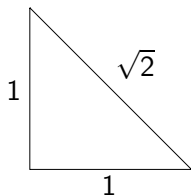
is rational and between a and b .

- ▶ Check!

Are all numbers rational?

- ▶ The Ancient Greeks thought they were.
- ▶ Pythagoras' theorem (580 BC): sides a , b , c of a right angled triangle satisfy

$$a^2 + b^2 = c^2$$



- ▶ But what about $\sqrt{2}$?

Assume that $\sqrt{2}$ is rational.

- ▶ Then we can write

$$\sqrt{2} = \frac{a}{b}$$

where a and b are integers, with no common factors. In particular, they are not both multiples of 2 (even numbers).

- ▶ Let's square both sides to make it simpler:

- ▶ and multiply up....

- ▶ Since $a^2 = 2 \times b^2$, then a^2 must be an even number.
 - ▶ So therefore a must be an even number.
 - ▶ So let's write $a = 2 \times c$.
 - ▶ Giving us...
-
- ▶ or, cancelling a 2 on each side

Hmmm....

- ▶ So now $b^2 = 2c^2$, so b^2 must be an even number.
- ▶ So therefore b must be an even number....

- ▶ *But we said a and b couldn't both be even numbers!*

- ▶ Contradition! $\sqrt{2}$ is *not* a rational number.

What about $\sqrt{3}$?

- ▶ Use the same ideas to prove that $\sqrt{3}$ is irrational.

Decimal expansions

- ▶ What does the decimal expansion of an irrational number look like?

$$\sqrt{2} = 1.414213562\dots$$

- ▶ What does the decimal expansion of a rational number look like?

$$\frac{1}{2} = 0.5$$

$$\frac{1}{3} = 0.33333333\dots$$

Periodic decimals

- ▶ Rational numbers have periodic decimal expansions:

$$\frac{1046}{495} = 495 \overline{)1046.0000 \dots}$$

A random number

- ▶ Pick a decimal number between 0 and 1 at random.
- ▶ What are the chances it is rational?

No holes, no neighbours

- ▶ Can you find an irrational number between two rational numbers? How can you find it?
- ▶ Can you find a rational number between any two irrational numbers? How?
- ▶ What's the rational number closest (but not equal to) zero?
- ▶ What's the irrational number closest to zero?

Important ideas from today:

- ▶ Some numbers are rational, others are not
- ▶ There are very few rational numbers compared to irrational numbers.
- ▶ But there are still infinitely many!

For next time

- ▶ Vivien Kirk will take over.
- ▶ Read Section 3.1 of the textbook. Beyond Numbers.