Maths 361: STUDY GUIDE FOR SEMESTER 1 2008

This is an introductory course in Partial Differential Equations (PDE's).We cover Fourier series, Fourier integrals, boundary value problems and separation of variables, with application to the solution of second order PDE's in one, two and three dimensions.

Preparation for the course: To succeed in this course, you need a good background in Stage II mathematics. In particular, you should have at least B- in Maths 253 and Maths 260 (Differential Equations) or in equivalent courses. If in doubt about whether you are adequately prepared for this course, contact the lecturer of the course.

Textbook: The text is Michael Greenberg's Advanced Engineering Mathematics, Second Edition. This book is now the text book for all Stage 3 applied mathematics courses in our department, and for Maths 340. Please see the lecture plan to find out which sections we cover in the book.

There are many other useful books about PDEs available in the library - look around! As a student you can download Tang's Mathematical Methods for Engineers and Scientists 3 through the university's Springer link. *Note that this link sometimes doesn't take you directly to the book the first time you click on it. If that's the case, come back to this page and try again.* This book covers most of the material in the course.

Lecturers:
First half of the course: Dr Steve Taylor (Room 306 Math, s.taylor@auckland.ac.nz)
Second half of the course: Prof Jari Kaipio (contact details to be announced later)

Lectures: Monday, Wednesday, Thursday and Friday at 10am. One class a week (usually Thursday) will be used for tutorials.

Course work: Four assignments will be set and marked. They form an important part of the course and should be attempted by all students. The assignments are due in on the dates: Thursdays 20 March, 10 April, 15 May, 5 June.

Mid-Semester Test: There will be one mid-semester test probably in the evening of 22nd April (*date to be confirmed*). It will be a one hour test.

All students should take this test. The room for the test will be announced in lectures and on this web site. If illness or other problems prevent you from completing any of the assignments please contact your lecturer as soon as possible. A medical certificate will be required if you wish to apply for an exemption from an assignment. If you are ill or have other problems at the time of the test or the exam you should contact Student Health and Counselling (extn 87681) immediately to obtain aegrotat information.

Final Mark: 20% assignments, 20% test and 60% exam.

Course Outline: Approximate number of lectures for each topic given in brackets. See the detailed lecture plan to find out how this relates to sections covered in the text book.

1. One-dimensional problems (3). Introduction to PDE's. Diffusion (heat) equation.

2. Fourier Series (5). Orthogonality of functions and sets of functions. Real trig series and even and odd functions.Convergence and sketching Fourier series.Complex Fourier series.

3. Boundary Value Problems (4). Eigenvalues and eigenfunctions. Sturm form and self adjoint operators. Sturm-Liouville eigenvalue problems, existence and orthogonality of solutions, eigenfunction expansions.

4. Laplace Transforms (3). Introduction to transform methods. Calculation and properties of the transform. Solution of ODEs.

5. Fourier Transforms (3). Fourier representation of delta function. Convolution theorem.

6. Application of Fourier and Laplace transforms to PDEs (2)

7. Wave Equation (9). Solution by Separation of Variables. Vibrating membrane. D'Alembert's solution for the vibrating string. Using Fourier and Laplace transforms.

8. Laplace Equation (7). Heat flow in 2D. Laplace's equation. Separation of Variables. Non-homogeneous boundary conditions. Laplace equation in polar, cylindrical and spherical coordinates.

Lecture Plan. Text: Michael Greenberg, Advanced Engineering Mathematics, Second Edition. The course covers material from the following chapters:

- 5. Laplace Transform.
- 17. Fourier Series, Fourier Integral, Fourier Transform.
- 18. Diffusion Equation.
- 19. Wave Equation.
- 20. Laplace Equation.

Lectures (3 per week for a total of 36)

1. **Chapter 18: Diffusion Equation**
   18.1 Introduction.
   18.2.1 Definitions.

2. 18.2.2 Classification.
   18.2.3 Diffusion Equation and Modeling.

3. 18.3.1 The method of separation of variables (up to Equation 25)

4. **Chapter 17: Fourier Series**
   17.1 Introduction
   17.2 Even, Odd, and Periodic Functions
5. 17.3.1 Fourier Series.
6. 17.3.2 Euler's formulas.
7. 18.3.1 The method of separation of variables (continued from Lecture 3).
8. 17.3.4 Complex exponential form for Fourier series.
9. 17.7.1 Sturm-Liouville problem
10. 17.7.1 Sturm-Liouville problem (continued)
11. 17.7.2 Lagrange identity
12. 18.3.3 Use of Sturm-Liouville theory
13. **Chapter 5: Laplace Transform**
    5.1 Introduction.
    5.2 Calculation of the Transform.
14. 5.3 Properties of the Transform.
15. 5.4 Application to the solution of differential equations.
16. **Chapter 17: Fourier Integral, Fourier Transform**
    17.9 Fourier Integral
17. 17.10 Fourier Transform
18. 17.10 (continued) Fourier Transform
19. **Chapter 18: Diffusion Equation (continued).**
    18.4 Fourier and Laplace Transforms.
20. 18.4 (continued) Fourier and Laplace Transforms.
21. **Chapter 19: Wave Equation**
    19.1 Introduction.
22. 19.1 (continued) Introduction
    19.2.1 Solution by Separation of Variables.
23. 19.2.1 Solution by Separation of Variables.
    19.2.2 Travelling Wave Interpretation.
24. 19.2.3 Using Sturm-Liouville theory
25. 19.3 Separation of Variables; Vibrating Membrane.
26. 19.3 (continued) Separation of Variables; Vibrating Membrane.

27. 19.4.1 Vibrating String; d'Alembert's solution.
28. 19.4.1(continued) Vibrating String; d'Alembert's solution.
29. 19.4.3 Solution by integral transforms.
30. **Chapter 20: Laplace Equation**
    20.1 Introduction.
    20.2 Separation of Variables.
31. 20.2 (continued) Separation of Variables.
32. Separation of Variables; Non-Cartesian Coordinates.
    20.3.1 Plane polar coordinates.
33. 20.3.1 (continued) Plane polar coordinates.
34. 20.3.2 Cylindrical coordinates.
35. 20.3.3 Spherical coordinates
36. 20.4 Fourier transform