Maths 260 Lecture 11

- Topic for today: More on bifurcations
- Reading for this lecture: BDH Section 1.7
- **Suggested exercises:** BDH Section 1.7, #11
- ▶ Reading for next lecture: BDH Section 1.8
- Today's handouts: None

Bifurcations

We are interested in one-parameter families of autonomous DEs, which can be written in the form

$$\frac{dy}{dt}=f_k(y).$$

We look for bifurcations, i.e., changes in the qualitative behaviour of solutions as the parameter k is varied.

General result about bifurcations

Bifurcations usually do not happen, i.e., a small change in a parameter usually leads to only a small change in the behaviour of solutions, with no overall qualitative change in behaviour.

To be precise, if

$$\frac{dy}{dt} = f_k(y),$$

and $\partial f/\partial k$ and $\partial f/\partial y$ exist and are continuous for all values of k and y, then a small change in k gives a small change in the graph of $f_k(y)$.

Example 1:

 $\mathsf{Suppose} \ \mathsf{the} \ \mathsf{DE}$

$$\frac{dy}{dt} = f_k(y)$$

has a source at $y = y_0$ when $k = k_0$, with

$$\left.\frac{df_{k_0}}{dy}\right|_{y=y_0}>0.$$

What is the effect on the qualitative behaviour of solutions of changing k by a small amount?

When do bifurcations happen?

A bifurcation in which the number or type of equilibrium solutions changes can only occur at $k = k_0$ if

$$f_{k_0}(y_0) = 0$$
 and $\frac{df_{k_0}}{dy}(y_0) = 0$,

This is when the linearization theorem does not work.

Example 2:

Draw the bifurcation diagram for the family of equations

$$\frac{dy}{dt} = ky - y^3$$

Example 3:

Draw the bifurcation diagram for the family of equations

$$\frac{dy}{dt} = ky + y^2$$

Important ideas from today:

- Bifurcations are special: a small change in parameter does not usually result in a qualitative change in the behaviour of solutions.
- A bifurcation at which the number or type of equilibrium solutions changes can only occur at k = k₀ if

$$f_{k_0}(y_0) = 0$$
 and $\frac{df_{k_0}}{dy}(y_0) = 0$,

i.e., at a parameter value where the linearization theorem does not work.