

Maths 190 Lecture 17

- ▶ **Topic for today:** The Dynamics of Change
- ▶ **Question of the day:** What do you need to know to predict the population of the world in the future?

Many physical systems have a “repeating” quality?

Examples from our world of things that repeat:

- ▶ Tides
- ▶ Motion of planets
- ▶ Rush hour

Iterative Dynamical Systems

A simple model of a bank balance

- ▶ Let B_n be the amount of money in a bank account at time n . If no withdrawals are made and the interest rate is 3% per year, then

$$B_{n+1} = 1.03B_n$$

- ▶ If the balance is initially $B_0 = \$5000$, then

$$B_1 =$$

Balance after 10 years is

Iterative Dynamical Systems

A really simple population model

- ▶ Let P_n be the 'population density' at time n . A population increasing by a constant percentage each year could be modelled by:

$$P_{n+1} = kP_n$$

- ▶ If $k = 2$ and $P_0 = 0.1$, then

$$P_1 =$$

A slightly less simple model

- ▶ A slightly less simple population model:

$$P_{n+1} = k(P_n - P_n^2)$$

- ▶ $k = 2, P_0 = 0.1$:

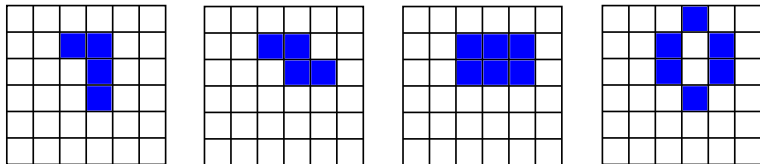
$$P_1 =$$

- ▶ $k = 4, P_0 = 0.1$:

$$P_1 =$$

- ▶ Often simple rules can have complicated behaviour.

The Game of Life

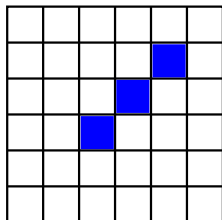
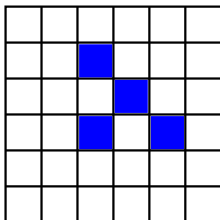
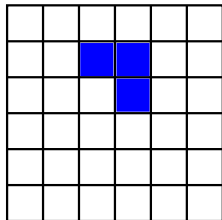


- ▶ The game is played on an infinite grid of 'alive' and 'dead' cells. Live cells are shaded, dead cells are blank.
- ▶ Each cell has eight "neighbours". The current status of a cell's neighbours determines the status (dead or alive) of the cell in the next generation.
- ▶ No random element in the game; just a few simple rules.

Rule 1: Staying alive

A living square will remain alive if it has two or three neighbours, otherwise it will die.

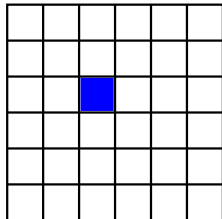
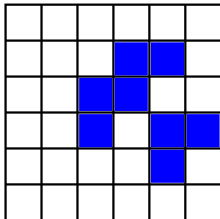
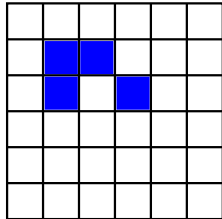
With the following initial configurations, which cells remain alive in the next generation?



Rule 2: Dying

A living square will die if it has fewer than two, or more than three neighbours.

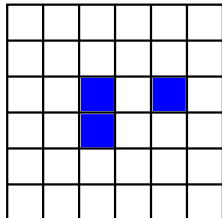
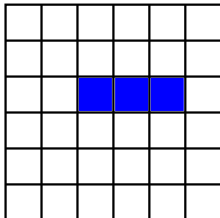
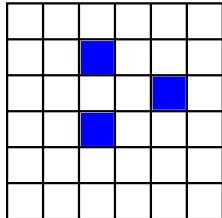
With the following initial configurations, which cells will die in the next generation?



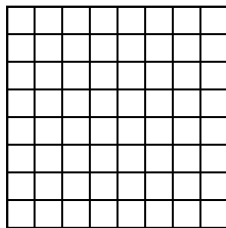
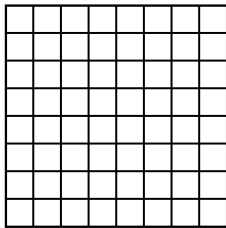
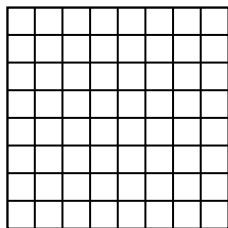
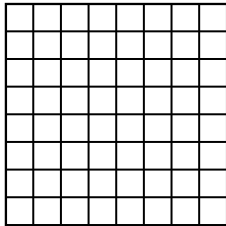
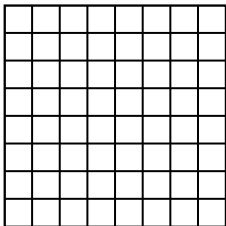
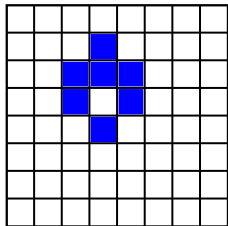
Rule 3: Coming alive

A dead square will come alive if it has *exactly* three living neighbours.

With the following initial configurations, which cells will come alive in the next generation?



Have a go!



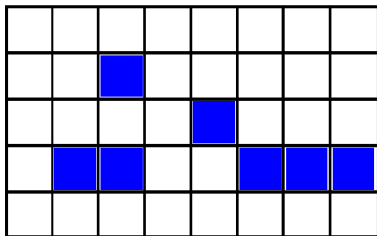
Types of behaviour

Types of behaviour we might see include:

- ▶ population explosion
- ▶ population extinction
- ▶ steady pattern
- ▶ periodic pattern
- ▶ migratory pattern

Types of behaviour

Sometimes the behaviour is very complicated. For instance, this initial configuration takes 5206 steps to settle down to a steady pattern.

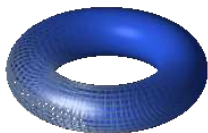


There's no way to tell from an initial configuration what will happen — just have to try it and see.

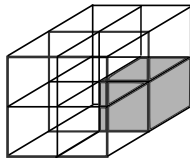
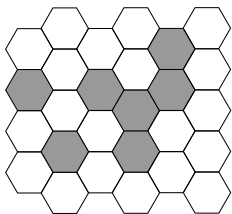
See <http://www.bitstorm.org/gameoflife/> for more information.

Other cellular automata

- ▶ Change the rules.
- ▶ Change the space.



- ▶ Change the grid.



Important ideas from today:

- ▶ The growth of populations can be modeled using fairly simple repeating rules.
- ▶ The Game of Life is one example of a simple population model.

For next time

- ▶ Read section 6.5