General Linear Methods

Maria Goodier

University of Auckland

Student Research Conference 2007

Supervisor: John Butcher

Abstract

Ordinary differential equations describe many types of real-life problems: population, mechanical, chemical and astronomical models. To predict the future behaviour of the quantities, described in such models, we have to solve a differential equation or a system of differential equations. Since only a limited number of differential equations can be solved analytically and exactly, we have to use numerical methods to find approximate solutions of many other types of differential equations.

Two well-known classes of numerical methods are Runge–Kutta and Linear Multistep methods. Runge–Kutta methods evaluate the right hand side of the differential equation at s points inside the current integration step but use only one initial starting value y_0 as input to a step. On the other hand, linear multistep methods use r multiple input values for each step but evaluate the right hand side only once. For these methods, the vector of multiple inputs has to be recalculated on every step.

General Linear methods generalize both Runge–Kutta and Linear Multistep methods. They allow for both s evaluations and r inputs per step. General Linear methods are potentially more efficient than other methods for solving exceptionally large systems of differential equations, for example, the thousands of equations arising in astronomical simulations.

For practical implementation of General Linear methods, it is convenient to use r-value Nordsieck input vector of scaled derivatives on every step. This special Nordsieck vector allows us to implement general linear methods with variable stepsize. It is shown, how the Nordsieck vector is updated on every computation step, and how the new stepsize is chosen to achieve the required accuracy with maximum efficiency.

General linear methods require starting methods to approximate the initial Nordsieck vector. It will be shown how to construct such starting methods using order conditions based on the known conditions for Runge–Kutta methods. Some solutions of these conditions and the resulting approximations for the initial Nordsieck vector will be presented.

The interpolation procedure for approximating the solution values at different time-points between the integration steps is described. An example of interpolation for the second and third order general linear methods is given.