On the order of G-symplectic general linear methods

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General linear methods for ordinary differential equations are multistage and multivalue methods. Multistage means that, like Runge–Kutta methods, more than one evaluation of the differential equation function is performed in each time-step. Multivalue means that, like multistep methods, more than one piece of information is passed between steps. We are interested in general linear methods for long time integration of differential equation systems having quadratic invariants. Runge–Kutta methods can preserve quadratic invariants if they are symplectic, meaning that the coefficients of the Runge–Kutta method satisfy an additional constraint known as the symplectic condition. We will explain how this generalises to the case of general linear methods. This give rise to the concept of G-symplectic general linear methods.

The order of accuracy, which expresses the asymptotic error behaviour of the method with small time-steps, requires a complicated analysis using the trees and the composition rules of B-series. In the case of symplectic Runge–Kutta methods, some of the trees disappear from the order conditions due to the symplectic condition. We will carry out an analysis to study the order of a particular G-symplectic general linear method.