DEPARTMENT OF MATHEMATICS

MATHS 190 Lecture 8 Summary

In this lecture we showed that ideas about infinity are often counter-intuitive.

Our main example was the ping-pong ball conundrum (page 149 of the textbook). This is a thought experiment lasting exactly 60 seconds. We start with a large empty barrel and a long line of ping-pong balls, numbered in order $1, 2, 3, \ldots$. We start the clock.

- In the first 30 seconds, put the first 10 balls into the barrel (numbers 1-10), find number 1 and throw it out.
- In half the remaining time (15 s), put the next 10 balls into the barrel (numbers 11-20), find number 2 and throw it out.
- In half the remaining time (7.5 s), put the next 10 balls into the barrel (numbers 21-30), find number 3 and throw it out.
- Continue in this way until 60 seconds has passed, then stop the experiment.

We saw (after quite a bit of thought!) that at the end of the 60 second experiment there are no balls left in the barrel.

In another example, we showed that the set of all fractions (rational numbers) was the same size as the set $\{1, 2, 3, \ldots\}$ of natural numbers. We did this by finding an explicit one-to-one correspondence between the elements in the two sets.

Before you come to the next lecture: You should spend an hour or two reviewing the material from todays lecture. You should also

- Read §3.2 in the textbook.
- Try some of the Mindscapes at the end of §3.2 in the textbook.
- Review the game of Dodgeball from Lecture 1 (Story 5 in section 1.1 of the textbook).

Other activities you could do if you have time are:

- Try to explain the ping-pong ball conundrum to a friend who is not in Maths 190.