Comparative Probability Orders and the Flip Relation

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A comparative probability order is a linear order on a set of subsets of a finite set X, with the conditions that $\emptyset \prec A$ for all $A \subseteq X$, and that if $A, B \subseteq X$ and $A \prec B$, then for all $C \subseteq X$ such that $C \cap (A \cup B) = \emptyset$, $(A \cup C) \prec (B \cup C)$. A useful application of comparative probability orders is in studying betting behaviour, where an agent believes some events are more likely than others and bets accordingly. Here events can be thought of as subsets of X, thus the agent's personal probability assessment is a comparative probability order.

The concept of the flip relation on comparative probability orders was introduced by Maclagan in 1999. Flippable pairs are defined as neighbouring pairs of disjoint subsets $A \prec B$ in a comparative probability ordering $\prec$ such that for every $C \cap (A \cup B) = \emptyset$ the subsets $A \cup C$ and $B \cup C$ are also neighbours in $\prec$. Flipping over a flippable pair $A \prec B$ gives rise to another comparative probability order in which $B \prec A$. Using this fact, the flip relation turns the set of orders on n elements into a graph $\mathcal{G}_n$, in which two orders are said to be neighbouring if one can be obtained from the other by flipping over a flippable pair.

We describe different representations of comparative probability orders, such as discrete cones and geometric polytopes, and explain what the flip relation represents in these contexts. We demonstrate a correspondence between flippable pairs and irreducible vectors of the cone, give an overview of the concept of (additive) representability, and show that cones for representable orders are generated by their irreducible vectors.

Finally, we conjecture that the maximal number of neighbours an order may have in $\mathcal{G}_n$ is equal to the $n + 1st$ Fibonacci number ϕ_{n+1}, and show this conjecture holds for $n \leq 6$. We can prove that in $\mathcal{G}_n$ there exists an order that has ϕ_{n+1} neighbours, and we conclude by presenting several open problems for further research.