

In memoriam

Günther Nürnberger  
January 13, 1948–May 11, 2013



Günther Nürnberger was born on January 13, 1948, near Marktredwitz, a little town in North Bavaria, Germany. In 1969 he matriculated in the university of Erlangen-Nürnberg and did there his diploma thesis in Functional Analysis in 1974. One year later he finished his doctoral thesis, both theses under the supervision of Dietrich Kölzow. In his thesis [1] he studied the problem of characterization, existence and uniqueness of (measurable, linear or continuous) selections for the metric projection in normed linear spaces.

In 1979, Günther received his Habilitation. From 1974 until 1983 he was an assistant resp. assistant professor at the Institute of Applied Mathematics in Erlangen. After this he held professorships at the universities of Mannheim (1983–1985) and Erlangen (1985–1989) and, finally, in 1989 he got a chair for Mathematics at Mannheim. He remained there until his untimely death. From 1993 to 1994 he was Dean of the Faculty of Mathematics and Computer Science. He served as an Associate Editor of six international journals, including Journal of Approximation Theory (since 1989) and Communications in Applied Mathematics (since 1997).

Günther was a very active and creative researcher. He wrote nearly 100 research papers (see the enclosed “List of Publications”). His book on “Approximation by Spline Functions” (Springer, 1989) has been often used as background for lectures and seminars on splines.

Günther’s contributions to approximation theory cover a number of popular subjects, such as continuous selections for the metric projection, uniqueness and strong uniqueness of best uniform approximations from univariate splines, best uniform approximation and interpolation

by generalized splines with fixed and free knots, algorithms for computing best spline and piecewise polynomial approximations, approximation and interpolation by bivariate splines on several types of triangulations, optimal approximation order, local Lagrange interpolation, interpolation by trivariate splines.

Günther was an international highly respected scientist and an excellent scholar. He was a kind, modest and hard-working colleague and teacher for his students and for all those who surrounded him. We hope that the following few pages of reminiscences, written by a few of his good friends and colleagues, give something of Günther's spirit.

## **Reminiscences of Günther Nürnberger**

**Hubert Berens** (*University of Erlangen-Nürnberg, Germany*)

Professor Berens asked us to say some words on his collaboration with Günther. For the first time he met Günther in October 1974 when he gave a lecture for the engineering students at the university of Erlangen. Günther was responsible for the exercise courses. At that time Hubert Berens formed a very productive research group, including Günther. Berens was very interested in Günther's research, especially in his studies on continuous selections of metric projections.

In 1983 a workshop on Approximation Theory organized twice the year at Erlangen resp. Eichstätt was initiated. Some years later after Günther had moved to Mannheim the workshop took place also at this university. Later on it had an expansion to some other universities in South Germany, such as Würzburg, TU München, Stuttgart-Hohenheim, Jena and Passau. Günther and his students gave many interesting talks on interpolation and approximation by uni- and multivariate splines at the workshop.

Hubert Berens and Günther published a joint paper [42] based on their collaboration on continuous selections. Especially, they showed that the metric projection of  $C[a, b]$  onto a space of polynomial splines is lower semicontinuous on an open, dense subset of  $C[a, b]$ . This implies that "standard" selections of the metric projection are continuous on the considered subset.

**Oleg Davydov** (*University of Strathclyde, Glasgow*)

My first contact with Professor Nürnberger was in 1995 when I approached him with a request to act as a host for a Humboldt Research Fellowship I was seeking to apply for while working at the Dnepropetrovsk University in the Ukraine. He responded enthusiastically and his suggestions helped to significantly improve my project plan. The application was successful and in June 1996 I arrived in Mannheim to take a four months German language course at the local Goethe-Institut. I immediately started working together with Günther and Frank Zeilfelder. After two months in Eichstätt and Erlangen with Manfred Sommer and Hans Strauss, I returned to Mannheim and spent further twelve months there.

Communication with Günther was always a great pleasure. He liked to put things into a general perspective, generously shared his ideas and was happy to discuss other people's thoughts. Most importantly, he was extremely enthusiastic about research in general and in particular about the problems of bivariate spline interpolation, the focus of his interest in these years, which encouraged me and everybody else who worked with him.

The Humboldt Fellowship has completely changed my life, as I remained in Germany for several years and later moved to Scotland. We stayed in a close contact for a number of years and co-authored seven research papers. I am very much grateful to Günther for the wonderful and decisive time in Mannheim. His ill-health in the recent years and premature death have shocked me tremendously. He will be remembered for his numerous mathematical results, a brilliant book on spline approximation and as a friendly and inspiring supervisor and collaborator.

**Frank Deutsch** (*The Pennsylvania State University, USA*)

Like others who knew Günther, I was shocked at the news of his death. People should not die at 65 years of age. I do not recall exactly where or when I first met Günther, but it was probably at an Oberwolfach conference in the early or mid seventies. He initially appeared to me to be quiet and unassuming. But later when I got to know him better, his more humorous and fun-loving nature stood out.

He spent a year's leave from Erlangen from September 1976 to September 1977 with me at Penn State, and we ended up publishing three papers [9,7], and [17] based on our collaboration there. (The paper [9], resp., [7] was also co-authored with Bruno Brosowski, resp., Ivan Singer, who each spent a few months with us in 1977 at Penn State.)

We had many pleasant times and meals together during his year at Penn State. We also had a few favorite hangouts in State College where we discussed mathematics and other things over a few beers. One of the many reasons I liked and respected Günther was that I never ever heard him say a negative word about anyone. Even in jest, he chose his words carefully in describing someone.

Günther was a big fan of Elvis Presley. He was saddened by Presley's death which occurred while Günther was at Penn State. I remember that for several days after Presley's death, Günther would repeatedly mention that "the king is dead, long live the king!"

During my sabbatical leave at the Universität Frankfurt in 1978–79, he invited me to give a talk in Erlangen and I visited him there for a few days. Günther knew that I was buying and collecting beer glasses with the names of various German beers inscribed. He arranged to have one of his friends who was a bartender to present me with a few beautiful glasses that I still have today. My guess is that either Günther paid for them beforehand without me seeing, or, perhaps more likely, the bartender liked and respected Günther so much that he gave them to me as a gift. In either case, that act of kindness sums up the kind of person that Günther was.

He wrote a very clear and beautiful book [101] in 1989 on Approximation by Spline Functions. It was so good that I used it in a special topics course I gave in the early 90s. I remember telling him that, in my opinion, the only thing that kept it from being "the perfect book" was that it did not have any exercises. Of course, I meant this as a compliment. (Has anyone ever written "the perfect book"?)

Over the years we met at many approximation theory conferences in both the United States and abroad. It was always nice to rekindle the friendship that we had started so long ago. Perhaps the last time we were together was in Nashville, Tennessee, at an approximation theory conference a few years ago. He seemed to be a bit disoriented, but I did not realize at the time that he was probably in the early stages of the disease that would ultimately claim his too short life. I will miss him.

**Paul Nevai** (*Columbus, Ohio, USA*)

Günther was one of the very few old-timers at JAT left over from the Shisha era. Although I met him several times at various conferences already much earlier, my first recollection of talking to him in person was on September 7, 1996, when I arrived to Mannheim to attend his multivariate conference. I was struck by his warm and down-to-earth personality.

After that, with the ubiquity of email, we stayed in constant touch regarding JAT related matters. I heard the gossip that Günther, like many of his compatriots, was a typical German professor in the sense that he never learned to use email and all his emails were read, written, and sent by his students (but not by his secretary as a genuine Herr Professor Dr, would do). To this day I do not know the truth about this but I was always careful not to write anything that would be inappropriate if other people read it such as *so and so is a total moron*, etc.

I saw Günther the last time in Eichstätt on July 24, 2009, when he approached me and told me in his usual extraordinarily kind way that he drove up (actually more like southeast) from Mannheim just to meet me. He seemed to be disoriented and I dragged him down for a short walk to the yard in front of the math department and then I took this picture of him.



After that he kept thanking me again & again and I kept telling him that I am the one who is grateful to him for posing for me. The last time I heard from him on a personal level was on September 18, 2009, when he emailed me: *“Thank you very much for the photograph. It was nice meeting you in Eichstätt and listening to your great talk”*.

I was saddened to hear about the Edinburgh incident in September of 2010 when he managed to give a talk but after that he became fully disoriented and needed help to get home.

Despite his medical problems he kept working as JAT’s associate editor almost till the end. The last paper he handled was JAT-D-12-00004 by S. Borodachov and T. Sorokina that he recommended to accept on July 10, 2012.

I miss you Günther.

**Larry L. Schumaker** (Vanderbilt University, Nashville, USA)

One of the great privileges of being a mathematician is the opportunity it provides to meet colleagues in their home countries, and to experience both their countries and their cultures. Sometimes such encounters lead to lifelong friendships. Such was the case with Günther and me.

I first met Günther at the University of Erlangen, either in the summer of 1978 when he was a Wissenschaftlicher Assistent, or in the summer of 1981 by which time he was a Privatdozent. In those days he was working on Weak Chebyshev Systems and Metric Projections, two topics that also interested me. We also shared an interest in splines. My 1966 Ph.D. Thesis dealt with approximation by splines, and I was very happy to find a group of people in Erlangen who shared my enthusiasm for the subject. In any case, these visits and another in 1983 led to a collaboration with Günther, Manfred Sommer, and Hans Strauss, which resulted in a series of four joint papers published between 1983 and 1989 dealing with generalized splines.

I saw Günther again in 1984 when I attended a conference on Approximation Theory in Mannheim which Günther, who by then had just moved to Mannheim, organized with Günter Meinardus. This proved to be the first of a series of opportunities that I had to visit Günther at the University of Mannheim, including four summers in a row in 2002–2005. Each time he was the perfect host, organizing hotels and transportation, but also making sure that we took some time

to enjoy the Mannheim summer. In fact, somehow, almost every time I visited coincided with the Mannheimer Stadtfest, and I have fond memories of strolling down Plankenstrasse and taking in all of the action. We enjoyed many a frische Haxen or a dozen Nürnburger Bratwürstchen together, not to mention more than a few beers.

Günther was also a very considerate host. I remember sitting in his office, often with Frank Zeilfelder, with a big stack of drawings of triangulations, and with the windows wide open. He did not want to miss any of the discussions, so when he felt the need for a cigarette, he would stand over by the windows since he knew that Frank and I did not care for the smoke.

Günther was also an eager participant in many conferences around the world. In particular, he attended a good many of the “Texas” meetings in approximation theory, where at times he helped by organizing minisymposia. He also attended our conferences in CAGD in France and Norway, and almost always contributed to the proceedings.

While Günther worked on other topics, he maintained a lifelong interest in splines, and in 1989 published a comprehensive book entitled *Approximation by Splines*. In the early 90s he got interested in multivariate splines, and in a paper with T. Riesinger initiated the study of Lagrange interpolation by splines on triangulations and quadrangulations. He wrote a long series of papers on the subject, and his enthusiasm for the topic eventually drew me into a second round of collaboration which resulted in a series of eight joint papers written with Frank Zeilfelder and in a few cases with one of his students or one of mine. These papers were published between 2001 and 2009. This was a subject he was still working on when I saw him at *Approximation Theory XIII* in San Antonio in March of 2010, and for the last time in Edinburgh in September of 2010.

I will miss you greatly Günther.

**Ivan Singer** (*Simion Stoilow Institute of Mathematics of the Romanian Academy, Romania*)

Günther loved his work and was a passionate researcher: usually wherever we met, on the street, at the cafeteria, etc., he started to talk about our research problems, new solutions, variants, etc. Günther loved jokes, was always smiling and ready to laugh. He loved light music, was always humming melodies. Günther loved football (soccer), he played in the local team in Erlangen and on every Monday and Tuesday he talked enthusiastically about his game of last Sunday. Our colleagues listened with interest at his stories about those games. I have fond memories of my visits at his family in Erlangen, his mother and his sister. When he moved to Mannheim, I visited him also there and saw that he formed there an enthusiastic research group.

For our joint work with him and Frank Deutsch and our good time at Penn State, see the reminiscence of Frank. For his activities in Erlangen see the reminiscences of Manfred Sommer resp. Hans Strauss.

**Manfred Sommer** (*Catholic University of Eichstätt-Ingolstadt, Germany*)

In April 1974 I met Günther for the first time, both of us young doctoral students at the University of Erlangen. We were colleagues there until 1982 and became good friends during this time. My first impression (and it does not change all the time) of Günther was that he had a strong passion for mathematics. He showed big enthusiasm and power when trying to solve a problem. His papers were always clearly written and his lectures and talks were well-prepared and understandable.

After having finished our dissertations in 1975, a long period of collaboration began. In our first joint paper [3] we gave a complete characterization of existence of continuous selections for the metric projection  $P_G$ ,  $G$  a univariate spline space. The methods used in the paper were of fundamental importance for our studies on metric selections in 1975–1980 and led finally

to a complete characterization of existence of continuous selections for  $P_G$  where  $G$  denotes a finite-dimensional subspace of  $C[a, b]$ .

The second focus of our joint research was the development of numerical algorithms for best spline approximations [19,20] and for segment approximation (joint work with Günter Meinardus and Hans Strauss) [33,39], respectively. Both studies were difficult and exhausting jobs, but especially thanks to Günther's optimism and energy we could come to a good end.

Another topic of our joint research was concerned with generalized splines. In a series of papers (joined work with Larry L. Schumaker and Hans Strauss) [21,24,26,40] we investigated problems of interpolation and approximation by generalized splines with fixed and free knots, respectively.

Günther and I spent a nice time on many international conferences. I especially remember our first common conference on Approximation Theory at Austin, Texas, in January 1976. For both of us it was the first travel by plane and we were very excited in the beginning. Later on, we enjoyed all impressions of the conference (where we met many leading experts in AT and got helpful suggestions for our future work). After the conference we made a round-trip to see some nice places in the West of USA.

Günther's big hobby was football (soccer). For many years he was an active player in football teams of the Sports Club Wacker Marktredwitz. During our common time in Erlangen, every Monday morning he told very detailed about his weekend match. I remember that, after having finished his dissertation in May 1975, he was very proud of an article in a local newspaper with the title "Fussballer wird Doktor".

In the last years I mainly met Günther at workshops on Approximation Theory which were organized by universities in the South of Germany (including Mannheim and Eichstätt). For the last time I saw him in July 2010, during a workshop at the University of Jena. Although he did not speak about it, I felt that he had some problems with his health. Nevertheless, it was a shock to me when I heard the bad news about his death.

**Hans Strauss** (*University of Erlangen-Nürnberg, Germany*)

I first met Günther Nürnberger when he came to the Institute of Applied Mathematics in Erlangen in the year 1974 after 5 years studying mathematics at this university. Under the supervision of D. Kölzow he received his doctoral degree from the University of Erlangen in 1975.

At the Institute of Applied Mathematics Günther became acquainted with spline functions. He also was interested in these function classes and began to work together with his colleague Manfred Sommer on this topic. In several joint papers Günther and Manfred have shown very important results concerning continuous selections of the metric projection, in particular for subspaces of spline functions.

Günther also extended some of my results concerning uniqueness of best  $L_\infty$ - and  $L_1$ -approximations for univariate spline functions in joint papers with I. Singer [15] and G. Meinardus [38]. We also have written several joint papers on generalized splines together with L.L. Schumaker and M. Sommer [21,24,26,40] and on segment approximation with G. Meinardus and M. Sommer [33,39].

In the year 1983 Günther left Erlangen and went to the University of Mannheim. He came back from 1985 to 1989 to Erlangen and then again moved back to Mannheim where he had got a chair in Mathematics. He stayed in Mannheim until his retirement in the year 2013.

Later I often have met Günther at workshops and international conferences. Several times we also were travelling after these conferences together with H.-P. Blatt and M. Sommer to interesting places—in particular in the US.

Günther liked football very much. He also was an active football player. Every Monday morning we were discussing the football results of the preceding weekend.

Günther was a very original researcher and has collaborated with many mathematicians. He was full of energy and has shown many important results. Moreover, he was a gifted teacher who could explain mathematics clearly and attractively. It was a pleasure to discuss with him mathematical problems. Günther was a nice colleague and friend. I will miss him.

**Guido Walz** (*Wilhelm Büchner Hochschule, Darmstadt, Germany*)

with contributions by **Thomas Rießinger** (*Bensheim, Germany*)

I met Günther Nürnberger the first time when he got his first professorship at the University of Mannheim in 1983. At this time, I was a student in the last year, and I was very impressed by this young and energetic young professor. He reinforced the team around Professor Günter Meinardus, who held the Chair of Applied Mathematics in Mannheim at that time.

One year later I got my diploma degree and became doctoral student of Günter Meinardus. In the following time I had many fruitful discussions with Günter Meinardus and Günther Nürnberger, as well, in which I learned a lot and got many new ideas.

Again one year later, in 1985, Günther Nürnberger left Mannheim to go back to the University of Erlangen, where he received a Professorship in Mathematics. But he always stayed in contact with the group in Mannheim, not only but mainly at the workshops in Approximation Theory, which at this time took place twice a year at the Universities of Erlangen, Mannheim, and Eichstätt.

In 1989, he was appointed as Full Professor for Applied Mathematics at the University of Mannheim, with the intention to succeed Günter Meinardus, who would retire a few years later. As we know today, Günther Nürnberger stayed in Mannheim for 24 years until his far too early death in 2013. During this time he was Dean of the faculty of Mathematics and Computer Science and Director of the Mathematical Institute. I remember many long and intensive discussions with Günther Nürnberger, Thomas Rießinger, Frank Zeilfelder, Klaus Freyburger and others, during which he never got tired but instead seemed to become more and more enthusiastic, although discussions could last until late evening hours. At all these opportunities, Nürnberger never pointed out that he was the Professor, but instead gave us the feeling that we all were scientists with equal rights in the discussion. Most of the time we talked about mathematical problems, mainly splines, but it could happen that he switched to problems in music, literature or education, where his enthusiasm was not less strong. Finally, he always brought one or more of us home by his car when he had the feeling it had got too late.

One very impressive experience for me was the Conference on Multivariate Approximation and Splines, which Nürnberger and I organized in 1996 in Mannheim, together with Jochen Schmidt from Dresden. Without Nürnberger's energy, optimism and organizing ability this would not have been possible; also the fact that he had contact to the leading experts in Approximation and Spline Theory all over the world was very important for the successful course of the conference.

After I had left the University of Mannheim in 1999, I was editor of an extensive encyclopedia of Mathematics. Needless to say that Günther Nürnberger supported me also in this project by writing some articles for it.

When I got the news about his unexpected death in May 2013 I was really shocked.

Goodbye, Günther Nürnberger. I hope you meet some nice guys in heaven, where you are now; tell them about bivariate splines, I am sure they will love it.

**List of doctoral students of Günther Nürnberger**

1. Klaus Freyburger (1991)
2. Martin Adam (1993)
3. Norbert Kohlmüller (2001)
4. Frank Zeilfelder (Habilitation, 2002)
5. Angelika Engelmann (2003)
6. Gero Hecklin (2004)
7. Andreas Dinh (2006)
8. Markus Rhein (2009)
9. Michael Matt (2011)
10. Georg Schneider (2013)

**List of Publications**

- [1] Dualität von Schnitten für die metrische Projektion und von Fortsetzungen kompakter Operatoren, Doctoral thesis, Erlangen, 1975
- [2] Schnitte für die metrische Projektion, *J. Approx. Theory* 20 (1977) 196–219.
- [3] (with Manfred Sommer) Characterization of continuous selections of the metric projection for spline functions, *J. Approx. Theory* 22 (1978) 320–330.
- [4] (with Manfred Sommer) Weak Chebyshev subspaces and continuous selections for the metric projection, *Trans. Amer. Math. Soc.* 238 (1978) 129–138.
- [5] Unicity and strong unicity in approximation theory, *J. Approx. Theory* 26 (1979) 54–70.
- [6] Nonexistence of continuous selections of the metric projection and weak Chebyshev systems, *SIAM J. Math. Anal.* 11 (1980) 460–467.
- [7] (with Frank Deutsch and Ivan Singer) Weak Chebyshev subspaces and alternation, *Pacific J. Math.* 89 (1980) 9–31.
- [8] Continuous selections for the metric projection and alternation, *J. Approx. Theory* 28 (1980) 212–226.
- [9] (with Bruno Brosowski and Frank Deutsch) Parametric approximation, *J. Approx. Theory* 29 (1980) 261–277.
- [10] R. DeVore, K. Scherer (Eds.), Strong uniqueness of best approximations and weak Chebyshev systems, in: *Quantitative Approximation*, Academic Press, New York, 1980, pp. 255–266.
- [11] (with Ivan Singer) E.W. Cheney (Ed.), Uniqueness of best approximations in spaces of continuous functions, in: *Approximation Theory III*, Academic Press, New York, 1980, pp. 687–692.
- [12] (with Hans-Peter Blatt and Manfred Sommer) E.W. Cheney (Ed.), Pointwise-Lipschitz-continuous selections for the metric projection, in: *Approximation Theory III*, Academic Press, New York, 1980, pp. 223–228.
- [13] (with Hans-Peter Blatt and Manfred Sommer) A characterization of pointwise-Lipschitz-continuous selections for the metric projection, *Numer. Funct. Anal. Optim.* 4 (1981) 101–122.
- [14] (with Dietrich Braess) Nonuniqueness of best  $L_p$ -approximation for generalized convex functions by splines with free knots, *Numer. Funct. Anal. Optim.* 4 (1981) 199–209.
- [15] (with Ivan Singer) Uniqueness and strong uniqueness of best approximations by spline subspaces and other subspaces, *J. Math. Anal. Appl.* 90 (1982) 171–184.

- [16] A local version of Haar's theorem in approximation theory, *Numer. Funct. Anal. Optim.* 5 (1982) 21–46.
- [17] (with Frank Deutsch) Weakly interpolating subspaces, *Numer. Funct. Anal. Optim.* 5 (1982) 267–288.
- [18] Strong unicity constants for spline functions, *Numer. Funct. Anal. Optim.* 5 (1982) 319–347.
- [19] (with Manfred Sommer) Alternation for best spline approximations, *Numer. Math.* 41 (1983) 207–221.
- [20] (with Manfred Sommer) A Remez type algorithm for spline functions, *Numer. Math.* 41 (1983) 117–146.
- [21] (with Larry L. Schumaker, Manfred Sommer and Hans Strauß) Interpolation by generalized splines, *Numer. Math.* 42 (1983) 195–212.
- [22] Strong unicity of best approximations: a numerical aspect, *Numer. Funct. Anal. Optim.* 6 (1983) 399–421.
- [23] C.K. Chui, L.L. Schumaker, J.D. Ward (Eds.), Strong unicity constants for finite-dimensional subspaces, in: *Approximation Theory IV*, Academic Press, New York, 1983, pp. 643–648.
- [24] (with Larry L. Schumaker, Manfred Sommer and Hans Strauß) Generalized Tchebycheffian splines, *SIAM J. Math. Anal.* 15 (1984) 790–804.
- [25] Global unicity in optimization and approximation, *Z. Angew. Math. Mech.* 65 (1985) T 319–T 321.
- [26] (with Larry L. Schumaker, Manfred Sommer and Hans Strauß) Approximation by generalized splines, *J. Math. Anal. Appl.* 108 (1985) 466–494.
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- [38] (with Günter Meinardus) Uniqueness of best  $L_1$ -approximations from periodic spline spaces, *J. Approx. Theory* 58 (1989) 114–120.
- [39] (with Günter Meinardus, Manfred Sommer and Hans Strauß) Algorithms for piecewise polynomials and splines with free knots, *Math. Comp.* 53 (1989) 235–247.
- [40] (with Larry L. Schumaker, Manfred Sommer and Hans Strauß) Uniform approximation by generalized splines with free knots, *J. Approx. Theory* 59 (1989) 150–169.
- [41] C.K. Chui, L.L. Schumaker, J.D. Ward (Eds.), On the structure of nonlinear approximating families and splines with free knots, in: *Approximation Theory VI*, Academic Press, New York, 1989, pp. 507–510.
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- [43] (mit Günter Meinardus) Lothar Collatz, *J. Approx. Theory* 65 (1991) 1–2.
- [44] (with Günter Meinardus, Thomas Rießinger und GuidoWalz) The work of Lothar Collatz in approximation theory, *J. Approx. Theory* 67 (1991) 119–128.
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