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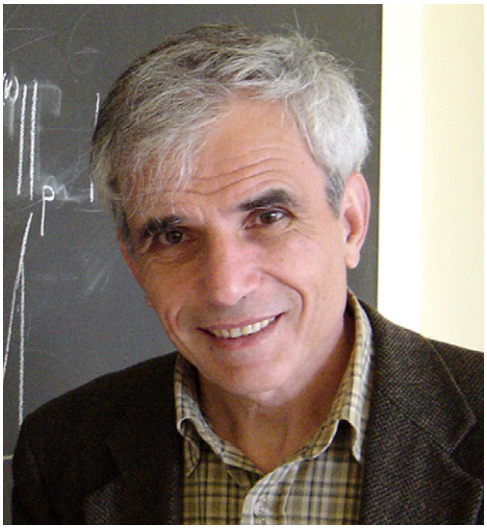
In Memoriam

Borislav D. Bojanov: 18 November 1944–8 April 2009



Borislav Dechev Bojanov was born in Shemshevo, a little village just outside the beautiful central Bulgarian city of Veliko Tarnovo, of which he was so fond. Borislav took his first degree at Sofia University. He then went to Wroclaw, Poland, where he did a Master's Thesis under the supervision of Professor Stefan Paszkowski. He returned to the Department of Mathematics of Sofia University in 1968 to serve as an Instructor, and stayed there until his untimely death. In 1975 he obtained his Ph.D. under the supervision of Professor Blagovest Sendov, and he was awarded his Dr. Sci. degree in 1981. Borislav Bojanov was elected a member of the Bulgarian Academy of Sciences in 1997. He also served in various administrative capacities, including that of Dean of the Faculty of Mathematics and Informatics from June 2003 to September 2007. Borislav was an active and important academic researcher, as is amply evident from the enclosed "List of Publications". He was an editor of this journal since 1994, and was also on the editorial board of six other journals. In 1995 he helped found the "East Journal on Approximations" and he was co-editor-in-chief thereof since its inception. But more than anything else, Borislav was a kind, optimistic, gentle, and wise person whom we had the privilege to call our friend. We hope that the following few pages of reminiscences, written by a few of his good friends and colleagues, and appearing in the order in which we received them, give something of the spirit of the man who had so much life in him.

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Reminiscences of Borislav Bojanov

Bl. Sendov (Bulgarian Academy of Sciences, Sofia). The extremely unexpected and untimely demise of Borislav Bojanov came as a shock to me. It was unbelievable. Borislav was so lively and healthy. I do not remember a single time when he was ill. He died in his sleep. A happy end for an old person, but not for a healthy man of 64. During my last vacation in Bulgaria, in the summer of 2008, we met several times and discussed the election of new members of the Bulgarian Academy of Sciences. He was responsible for managing the Mathematical Sciences in these elections. Of course, we discussed many general problems relating to the situation in Bulgaria and the future of our country, especially that of science and education. Borislav was not happy with this situation. He was trying to help by tackling responsible positions, but unsuccessfully. He was disappointed by his failure to compete for the position of Rector of Sofia University, due to artificial age limitations. He was also an unsuccessful candidate for President of the Bulgarian Academy of Sciences. In both cases he would have been a very good choice. As usual, Borislav was smiling and joking about these failures, but probably deep inside him there was a pain.

It is impossible for me to now remember when I first became acquainted with Borislav. He started his undergraduate education in the Faculty of Mathematics of Sofia University in 1962 and after 1965 continued his education in Poland, where he graduated from the University of Wroclaw in 1968. The same year I returned from a one-year specialization in Imperial College, London, and became professor of computer science at Sofia University. From that year I have a clear memory of Borislav as a member of our faculty. He became part of our group working in approximation theory and quickly earned the respect of his colleagues. As my Ph.D. student, Borislav started his successful study of extremal problems in approximation theory. We all know that the extreme is beautiful. It was my privilege to write about his scientific achievements every time he was promoted, from assistant professor to member of the Bulgarian Academy of Sciences. At every such occasion I was happy to evaluate his talent and unique achievements. Today, unfortunately, I have to write for his obituary. I feel like a father who has lost his son.

Bulgaria has lost not only one of its most talented mathematicians. Bulgaria has also lost one of its most devoted citizens. Borislav was a great patriot and was working hard for the benefit of our country. As Dean of the Faculty of Mathematics and Informatics of Sofia University, Professor Borislav Bojanov made significant progress in improving the conditions for education in a difficult political and economic environment. His creation of the East Journal on Approximations is an example of his drive to promote Bulgaria as an important country in mathematics.

Guergana Petrova (Texas A & M, College Station). I first met Borislav Bojanov during my second year as an undergraduate student at Sofia University, where he taught the Numerical Analysis class I was attending. I was impressed by this professor, who always came to class well prepared, and flawlessly and smoothly presented elegant proofs to difficult problems. The exam was quite challenging and for an "A" one had to solve additional problems where not only knowledge but also mathematical intuition and ingenuity were necessary to complete the proofs. Then, during my third year, following the program requirement, I had to select a particular concentration area of mathematics which would determine my further specialization. I did not hesitate and chose "Numerical Methods and Algorithms" as my core area and him as a professor to work with. At that time I did not realize that I had made one of the most important decisions in my career as a mathematician.

My work with Borislav Bojanov centers around the explicit construction, and the questions of existence and uniqueness of Gaussian quadrature formulae that use as recovery information various functionals of the function. Borislav Bojanov was the main driving force behind our joint work. Among other things, we discovered the unique quadrature for computing the integral of a function over a disk that is based on line integrals over n line segments and integrates exactly all bivariate polynomials of degree $2n - 1$. Our first joint manuscript turned out to be my first mathematical paper. We continued our collaboration through the years and ended up writing four papers together, of which the last one was finalized just days before he passed away. We were in the process of preparation of another manuscript which we were planning to finish this summer.

Being first my Master's Thesis advisor, then a mentor, colleague and friend, Borislav Bojanov had an enormous impact on my education and my life. Without his encouragement to pursue a Ph.D. degree, his constant help, support and guidance, I would not be the person and mathematician I am now. I am really grateful that I had the good fortune and the privilege to know Borislav Bojanov and to work with him. And I am deeply saddened that a mathematician of such talent, capability and kindness is not among us any more. His sudden, unexpected death came as a shock and a reminder that in this busy life we need to show our appreciation and cherish the people we value while they are still alive. I have lost a role model, a mentor and a friend, but his life's work remains, as does the mark he has made on the mathematical community and the people who knew him. I commemorate him with respect and gratitude, and will treasure his memory forever.

A. Zhensykbaev[1] (Institute of Mathematics, Almaty). Borislav Bojanov was an excellent mathematician. He made outstanding contributions to the theory of functions. His research interests were wide and varied: optimal recovery (in particular quadrature formulae), polynomial inequalities, and spline approximation. I want to discuss his research that was within our common scientific area of expertize.

I met Borislav for the first time in 1975 at the International Conference on Approximation Theory held in Kaluga. At that time he was doing research on the Kolmogorov–Nikol'ski problem with regards to best quadrature formulae on Sobolev classes of functions, i.e., its existence, uniqueness and properties. He had proved the existence of optimal quadrature formula on the class W_p^r, and that among all quadrature formulae with n nodes the optimal one has n distinct nodes (no multiplicities). These results were published in several good journals; see [28, 37]. I also very much liked the fact that in his papers Bojanov recovered and popularized many results connected with the theory of quadrature formulae of previous older Bulgarian mathematicians (Chakalov, Obreshkov). Problems of existence of optimal quadrature formulae are connected with monosplines of least deviation. This was researched by Bojanov and published in [31, 32].

At our first meeting in 1975 we started a friendship of many years. We frequently discussed our research problems in the theory of quadrature formulae, monosplines, and spline approximation. By the 1980s both of us had defended our Doctoral Theses on these topics. After that we saw each other many times at conferences on approximation theory in Bulgaria, Russia, Poland, Hungary, Yugoslavia, etc. And every time we organized a meeting with some of our old friends. Borislav was a pleasant, cheerful and reliable person, and the life and soul of any party.

In 1992 we wrote a joint paper on the existence of Gaussian quadrature formula for weak Chebyshev systems; see [75]. It was known that a quadrature formula with n nodes could be

[1] Sadly Professor Zhensykbaev passed away in September, 2009.

exact on a weak Chebyshev system of order at most $2n$. The question was: Can we always find a quadrature formula with n nodes which is exact on a given weak Chebyshev system of order $2n$? This had been positively solved for Chebyshev systems. Now we solved this problem for arbitrary weak Chebyshev systems. Some interesting papers of Borislav are dedicated to comparison-type theorems. If two Birkhoff-type quadrature formulae use the same number of nodes, counting multiplicities, which is better on a given class of functions? In short, the answer is that the formula is better if it uses values of derivatives of lower order; see [71, 72]. His later researches into numerical methods were dedicated in particular to cubature formulae. He approximated integrals over a circle by linear combination of integrals over parallel chords; see e.g. [103, 105].

Borislav used in his research many original methods from the theory of functions, algebra, and topology. He was a very creative mathematician. It saddens me that such an outstanding, excellent mathematician has died, at such a relatively young age.

Henryk Woźniakowski (Columbia University and University of Warsaw). On 8 April 2009, I received an e-mail from Geno Nikolov with the shocking news that Borislav Bojanov had unexpectedly passed away. I could not believe that my dear friend Borka, as I used to call him, was no longer with us. Even today it is very hard for me to cope with his death. I vividly remember him, his smiling face and his gentle personality. I still expect to see him and discuss one more mathematical problem.

Probably the best way to cope with the loss of Borka Bojanov is to remember the many nice meetings that I was fortunate to have with him. I met him for the first time in the early 1980s. You may know that Borka studied mathematics in Poland, in Wrocław. As a byproduct of his stay in Wrocław, Borka learnt Polish very well, and we mostly spoke in Polish. I addressed him using his diminutive first name Borka, and he addressed me using my diminutive first name Heniek. In the 1980s I started my interest in optimal algorithms and optimal recovery, and it was Borka who convinced me about the importance of Smolyak's powerful result. This result states that a linear algorithm minimizes the worst case error in the class of all (non-linear) algorithms using finitely many function values at prescribed points for approximation of a linear functional over a balanced and convex class. At that time Borka had already written a number of beautiful papers on integration and interpolation where he used Smolyak's results as the initial step in his search for optimal points of function values.

Borka visited me a couple of times at Columbia University in New York. Sometimes his visits were short, but once he spent a month at Columbia, I think it was around 1985. This was a remarkable visit mathematically and non-mathematically. I will restrict myself to two non-mathematical episodes.

Columbia promised to cover Borka's travel expenses for the round trip from Sofia to New York. I still remember a question from our secretary at Columbia who had trouble converting Bulgarian levas to American dollars. I suggested calling a local bank to find the current exchange rate. Some days later Borka entered my office with a check in his hand and a curious smile on his face. He showed me the check to reimburse his travel expenses and the check was for . . . $106,000 dollars (yes, for 106 thousand dollars). Well, it was just a simple math error—a factor of 100; instead of $1,060 dollars, somebody issued a check 100 times bigger. To the surprise of both of us, Columbia's administration was somehow not quite happy when the check was returned by Borka. Probably the pain of seeing a mistake was larger than the gain of almost $105,000.

The second episode was at the end of his stay, when I drove Borka to Kennedy Airport. At that time we had many visitors at Columbia, and maybe not so surprisingly, many of them were

from Poland. Obviously, Borka spoke Polish with all these Polish guests. When it was time to say goodbye, Borka mentioned that he had hoped that one result of his visit to Columbia would be that he would improve his English—instead, he had improved his Polish.

I also remember two more short meetings with Borka. When he finished his stay in Texas, he drove with his wife and two sons to New York. He did not know what to do with his car. He was quite glad to learn that my graduate student Tom Jackowski was looking for a used car and was happy to buy this car from Borka. In fact, Tom had this car for many years during his Ph.D. study and the car worked very well. Finally, I think it was in 1999 when I visited Texas A&M University, when Borka was spending a year there. I still remember that he took me to a museum and bought me a book about American presidents. This book still stands on a shelf in my Warsaw apartment.

There were many more meetings with Borka in Bulgaria, Germany and Poland. We edited together a book *Optimal Recovery*, based on talks during the conference "Open Problems in Approximation Theory" organized by Borka in Voneshta Voda, Bulgaria, in 1993. We often talked about writing a joint paper, but I am sorry to say that it unfortunately did not happen. And now it is too late to eliminate this mistake.

Farewell my dear friend. I will always cherish many beautiful meetings with you. I and many of your friends will miss you very much. We will always remember you and your work.

Zbigniew Ciesielski (Instytut Matematyczny PAN, Sopot).

Dear Borislav,

I realize that writing this letter is an irrational crossing of the boundary of time. Perhaps a justification for doing this may be our special friendship which became apparent to me too late, only after you had passed away. I believe that your friendly relations with Poland started with your graduate studies in Wroclaw University where you obtained in two years your Master's Thesis with Prof. Stefan Paszkowski. Let me recall that two years later we met for the first time at the Varna 1970 conference on *Constructive Function Theory* and the last time during the 2004 conference *Approximation and Probability* organized in Bedlewo on my 70th birthday. That was also the same year that your friends and colleagues celebrated your 60th birthday. During these almost 40 years we met at various conferences on approximation theory in Europe and in the USA. It may be proper at this point to mention, I hope you do not mind, that in our life we were lucky to have exceptional mathematician friends such as Hakop Hakopian and the late Vasil Popov, and they remain in our memories for ever.

Mathematicians know that mathematics is saturated with difficult extreme problems. Certainly you knew this very well. I know that you are highly respected for having solved long standing classical extreme problems in cubature formulae, algebraic polynomials and various approximation questions.

During 2003, when you visited the Mathematical Institute of the Polish Academy of Sciences in Sopot, we started to work on biorthogonal systems of polynomials corresponding to progressive finite differences placed at equally spaced points in [0, 1]. The manuscript is still waiting. Now I have the motivation to look into it seriously and, with your permission, to do something sensible with it.

Founding the East Journal on Approximations in spite of all the difficulties, and running it successfully as the Editor-in-Chief for 15 years, is your great contribution to science and in particular to mathematics, and this should be recorded in the history of science.

I am looking forward to meeting you again. Zbigniew

Raytcho Lazarov (Texas A & M University, College Station). I first met Borislav (Borka) in 1965 when he came to Wroclaw University after two years of studying mathematics at Sofia University. I was one year older and I quickly introduced him to university life in Poland. From the very beginning he struck me with his wit, big sincere smile, and mathematical talent. We became good colleagues and friends. We both admired our professors Steinhaus, Knaster, and Marczewski, all from the Banach group that moved after World War II from Lvov to Wroclaw. It was always a great pleasure to be in his company. We had memorable outings together to Prague during our vacations or long weekends.

Borislav returned to Bulgaria after graduating from Wroclaw University with an MS degree. At that time I was already working at the Centre for Research and Education, a dynamic and rapidly expanding joint organization between the Bulgarian Academy of Sciences and Sofia University. I was convinced that this would be a good place for Borislav, so I talked about him to Prof. Sendov, head of the Department of Mathematical Modeling at the Centre. Sendov understood very quickly the great value of this young talented man and used his influence to get a position for him at Sofia University. In this way the university got a remarkable mathematician and faculty member, and I got a friend for life. We were again together abroad, when during my Ph.D. study at Moscow State University he came for a one-year specialization under the supervision of N.S. Bakhvalov. During his stay at Moscow State University he started working on optimal recovery of functions and functionals, where he made major contributions. Borislav was a talented mathematician and remarkable problem solver. He proved the famous, open for more than 40 years, conjecture of Erdős, an unsurpassed problem solver, about the longest polynomial. His scientific research spans a vast variety of topics in classical analysis. He completely solved the problem of existence, uniqueness and characterization of optimal quadrature formulae with multiple nodes in the Sobolev spaces setting and generalized the classical Gaussian quadrature formula for extended and weak Chebyshev systems. He proved beautiful comparison theorems for various recovery schemes based on information of the same cardinality. He was among the first to apply the topological degree of a mapping to proving the uniqueness of solutions to various problems in approximation theory. He made essential contributions to the theory of spline functions, investigating B-splines, and σ-perfect splines with Hermite- and Birkhoff-type data. Some of his research represents new, alternative insights to classical results in approximation theory, such as the Hobby–Rice and Gauss–Krein theorems, the Jackson theorem and its extension to Chebyshev systems, the Remez inequality, the Bernstein comparison theorem, and many others. He gave brilliant proofs to several classical problems, and obtained beautiful extensions of the classical Markov inequalities for L^p-norms, and for polynomials with multiple zeros. He solved Favard's problem about the optimal Hermite interpolation of functions in Sobolev spaces. And the list goes on and on.

He was a magnet for young people studying mathematics at Sofia University. He attracted them with his ability to clearly formulate interesting and exciting mathematical problems and with his great appreciation for the beauty of mathematical proofs. No wonder he was able to recruit the best of them, guide them in their MS and Ph.D. studies, and mentor them as young faculty members.

As a Dean of the Mathematics and Informatics Department at Sofia University he showed a great vision for mathematics and informatics and their broader impact on other sciences. For example, under his leadership three new majors, Computer Science, Software Engineering, and Information Systems, were created; the colloquium of the Department was resurrected; and the connections with companies and business sponsorship were substantially improved and extended. And most importantly, he was able to instill a spirit of common goals, shared

governance, and togetherness of the student and faculty bodies. Students admired him and his colleagues loved him.

In 1986 Borkata was about to leave for a one-year visiting position at Texas A & M University. He was saddened by the fact that the Bulgarian authorities did not allow him to take his entire family with him; he had to leave their youngest son in Sofia. Nevertheless he was excited about the unbelievable new situation: the Berlin wall was still standing high, but the "wall" between East and West had been cracked and there were opportunities for travel never possible before. He was among the first to realize that the world was opening up and it was up to us whether we could use this new opportunity. He encouraged me to seek a visiting position in the USA. This happened in the fall of 1987 and changed my professional and personal life forever.

We shared common family values, a passion for mathematics, and we viewed the world in quite similar ways. We were both outside the political parties and social circles in communist Bulgaria of the 1970s and 1980s. For a period of eight years we lived in the same neighborhood of Sofia, and our families became very close. His three sons, who are now fine men and good citizens, and his dedicated wife and loving mother, became dearest friends to me and my family. At our gatherings we enjoyed good things in life and had great times. Sadly he passed away just before we realized our grand plan for this summer — visiting the old monasteries in the vicinity of his native town, Veliko Tarnovo.

Borislav dreamt that after the political changes in the early 1990s his beloved Sofia University would become a leader in science and education in the country and a place with political independence, economic freedom, and true academic spirit. But he came to realize that for demographic, social, and political reasons, education was in decline and the university was still very bureaucratic. This was breaking his heart. Recently his voice was regularly heard in support of teachers, scientists, faculty, and students; he was fighting the establishment for more resources for the universities, for higher recognition for scientists and faculty, and for making education a priority for Bulgaria. He was very much disturbed by the lack of such progress, one of the few things he never accepted in the new "democratic" Bulgaria.

Borkata was the most patriotic man I have ever known. Brought up in a family with roots in the Bulgarian movement for political liberation from the Ottoman Empire in the 1870s and having a father priest, he had inherited high spirituality, rare love for his country, and passion for the Bulgarian people. He never stopped admiring his countrymen, the beauty of the Bulgarian nature, and the rich history of our nation. After I moved to the USA he would often ask me "Doesn't your heart bleed for Bulgaria?"

His heart stopped beating on 8 April 2009 and I lost my friend of 44 years! He left this world at a time when he was needed by the Bulgarian mathematical community, his friends, and his family very much.

Borka, my dear friend, I miss you very, very much!

Geno Nikolov (Sofia University). My first acquaintance with Borislav Bojanov was in 1981. At that time I was a second-year math student at Sofia University, and I was attending his course in Numerical Analysis. I remember well my very first impression, when a young, smiling, energetic man entered the classroom, introduced himself, and soon after filled in with even handwriting the whole blackboard, even before some of my fellow students managed to open their notebooks. Amazingly, this speed did not affect in any way the brilliance of his presentation and the clarity of his arguments. Although by that time I had met several very good lecturers, I put Borislav Bojanov at the first position among them. Since then my ranking list of lecturers has significantly expanded, but its top has not changed.

Naturally I chose in 1984 Borislav Bojanov as advisor of my Master's Thesis, and later, in 1986, he became advisor of my Ph.D. Thesis. Both were devoted to the comparison of quadrature formulae of Birkhoff type, and revealed the intuitively clear fact that information consisting of point evaluations of lower-order derivatives is better for the recovery of integrals. Our first joint paper was about the existence of Birkhoff-type Gaussian quadrature formulae, and I would like to quote here its main result, as this was one of my first contacts with the beauty of mathematics. Here is the result: Given n non-negative integers $k_1, k_2, \dots, k_n$, satisfying some natural assumptions, there exists a quadrature formula using values of the derivatives of the integrand $f^{(k_i)}$, $i = 1, \dots, n$,

$$Q(f) = \sum_{i=1}^{n} a_i f^{(k_i)}(x_i),$$

which evaluates exactly $I(f) = \int_a^b f(x)\,\mathrm{d}x$ whenever f is an algebraic polynomial of degree not exceeding $2n-1$. Moreover, the nodes $\{x_i\}$ minimize $I(\Omega)$, where Ω is the unique monic algebraic polynomial of degree $2n$, satisfying the Birkhoff interpolatory conditions $\Omega^{(k_i)}(x_i) = \Omega^{(k_i+1)}(x_i) = 0$ for $i = 1, \dots, n$. One can easily recognize the classical Gaussian case when $k_1 = k_2 = \cdots = k_n = 0$.

Borislav Bojanov always tried to convey to us, his students, his refined taste for beautiful mathematical problems. He didn't like the attempts of various authorities to introduce priorities in mathematics, in particular declaring the superiority of applications to problems of pure mathematics, focusing on investigations (and financing) in a specific field of mathematics. He believed that such politics killed the charm of mathematics; it was like writing poetry on request. And the result would be the production of a huge number of publications that will be forgotten within a short period of time. Bojanov used to say that in mathematics one could be remembered for a single masterpiece, but also could remain unnoticed even with hundreds or thousand of papers. And he used to tell us a story about (I think) Stechkin, who, standing in front of Michelangelo's "David", said: "Poor Michelangelo, in a few centuries your precious work of art will no longer exist, while my theorems will last forever".

The evaluation of Professor Bojanov's scientific heritage would be an ambitious task that I could hardly accomplish, but amongst his papers there are some real gems. Just to give an idea, I will mention only one of his brilliant observations. If f is an oscillating algebraic polynomial of degree n in an interval $[a, b]$, then f is uniquely determined (within a factor ± 1) by the magnitudes of its local extrema $h_0(f), h_1(f), \dots, h_n(f)$, where the values at the end-points are included. Jointly with Professor Qazi I. Rahman from Université de Montréal, Borislav Bojanov showed that each of the corresponding quantities for $f', h_0(f'), \dots, h_{n-1}(f')$, depends monotonically on $h_0(f), h_1(f), \dots, h_n(f)$. This observation appeared to be an important tool for the derivation of various polynomial inequalities of Markov and Turán type. Using this technique, Bojanov proved a long-standing conjecture of Paul Erdős about the "longest" algebraic polynomial. He extended the monotonicity property to oscillating trigonometric polynomials and splines, and applied it to obtain further inequalities of Markov and Turán type.

I remember my first trip to Western Europe in 1992, when Professor Bojanov and I got invitations to attend a conference on numerical integration at the Mathematical Institute of Oberwolfach, Germany. On the way to Oberwolfach we decided to spend a night in Heidelberg. We took a walk in the old town, talking about various subjects and enjoying the nice November evening. However, I felt he was seeking some particular place in the town. It took him an hour or so until he said "Here it is!", and we entered a small bistro. He ordered beer for us, and revealed

to me that on his last time in Heidelberg he was led to that bistro by Vasil Popov (who had passed away two years earlier, in 1990). He said: "Let us commemorate Vasil. He was an outstanding mathematician and a great person. May he rest in peace."

In our conversations Professor Bojanov always showed his deep respect for the great Bulgarian mathematicians. He talked with admiration to us, his students, about the pioneers of Bulgarian mathematics, Academicians Lubomir Chakalov and Nikola Obreshkov. I remember the big portrait of Obreshkov standing in front of Bojanov's desk in his small office in the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. Bojanov ranked Obreshkov's monograph "Zeros of Polynomials" amongst the best mathematical books ever written. When I was a first-year Ph.D. student I remember turning over the pages of Bojanov's own copy of Obreshkov's monograph, and finding a page-marker made from a cloth strip with the colors of our national flag. Now Obreshkov's monograph has been translated into English, and thanks to Bojanov's efforts, two volumes with selected papers of N. Obreshkov were published. We learnt from Bojanov that many of the basic properties of the univariate B-splines, usually attributed to H.B. Curry and I.J. Schoenberg, were obtained about ten years earlier by L. Chakalov. This fact became known to the mathematical community thanks to the paper of Carl de Boor and Allan Pinkus, *J. Approx. Theory* **124** (2003), 115–123. In 2006, Professor Bojanov organized the international conference "Pioneers of Bulgarian Mathematics", dedicated to the 110th anniversary of N. Obreshkov and the 120th anniversary of L. Chakalov. During his term as Dean of the Department of Mathematics and Informatics of the Sofia University, he raised money to improve the Faculty building significantly. One of his acts was to put monuments of Obreshkov, Chakalov, Tzenov, and Petkanchin on the first two floors, as well as to put the portraits of all math professors that had lectured in the University on the newly built fifth floor.

Professor Bojanov was an eloquent story-teller. He used to cheer up company by telling stories and jokes. Here is a typical piece from his repertoire: During the communist regime, Kiril Dochev (a Bulgarian mathematician) had to pass some test on an ideological subject (called in those days "scientific communism"). The head of the committee asked him the first question, and Dochev gave his answer starting with "According to Stalin ...". And to the second question Dochev replied with "As Lenin wrote in his paper ...". The head of committee got annoyed: "But don't you have your own opinions on these issues, to not only quote Lenin and Stalin ?" "I have – Dochev said – but I do not agree with them ...."

The week before his death, Professor Bojanov and I were working on the preparation of an international research project with the Swiss Science Foundation. We met also on 7 April 2009; he looked well and he was planning a trip for the next day. And on the next day the terrible news came. He passed away in his sleep.

For various reasons, I was the student of Professor Bojanov who had the privilege to spend the most time with him, being first his MS and Ph.D. student, and then his colleague in the Section on Numerical Methods and Algorithms in the Department of Mathematics and Informatics of Sofia University. During that period I always felt his whole-hearted support, encouragement and friendship.

One never knows what surprises the next day hides. I wish we had spent more time together. I will miss our regular Thursday tea-meetings in his office where we discussed mathematics, politics, and life. Our seminars on Approximation Theory will never be the same without him.

It is sad that my words of respect, admiration and worship for my Teacher come at a moment when he is no longer with us. I wish I had expressed my appreciation to him when he was still alive, and I hope he did realize it.

I will treasure his memory forever.

Dimitar K. Dimitrov (Universidade Estadual Paulista, São José do Rio Preto, Brasil). Professor Borislav Bojanov was my teacher since 1985. He not only used to teach mathematics and doing research, but also educated young students to be correct, honest, modest and self-confident at the same time. From what I remember, Gabor Szegő said the only way to teach someone to do research in mathematics was by example, by simply doing research. Bojanov taught and educated his students this way. He was an example to me and to all his former students.

I remember very well the first time I met him and talked to him. He had published a survey paper "Sendov's conjecture on critical points of polynomials", in Bulgarian; see [57]. I was a second-year undergraduate, and a former high school teacher of mine who knew Bojanov and Sendov gave me his copy of that issue with the paper. I read the statement of the conjecture that night. I had already done a course on higher algebra which was a modern version of the course designed by Obreshkov so, together with topics like Silov groups and Galois theory, I had learnt something about polynomials and their zeros. I was fascinated by the conjecture and spent a year reading, trying to understand it well and, I confess, prove it. So, a year later, when I had to ask someone to supervise my Master's Thesis, I had no doubts, and one day in September 1985 I approached Bojanov at the entrance of the mathematics department building asking him to be my advisor. He stopped, looked at me and said he didn't remember me, what was my average mark, who taught me Numerical Analysis and my mark on that subject. When I said I had done it with Vasil Popov and had a high mark, he immediately agreed and began climbing the stairs to the second floor. But I followed him and said I wanted to work on Sendov's conjecture. He stopped in the middle of the stairs, looked at me again, smiled and said it was a bit risky because I might not prove it within two years and we ought to try learning and working also on other interesting and difficult problems. Probably most of the readers know that the conjecture still remains open and the last published paper of Bojanov is another survey, see [129], entitled "50 years of Sendov's conjecture".

I remember his small office at the Institute of Mathematics during the years when I was doing my Master's and Ph.D. Theses, where sometimes three or four of us would have tea and talk about mathematics, politics, history, and literature until late in the evening. Obreshkov was looking at us from a big portrait. When we switched to maths Bojanov would clean a small area of the blackboard and draw a graph or a picture, and say something about it, but never go into small technical details. When I was trying to describe details concerning a problem I was working on, after listening to me patiently for a long time he would ask what was the "target" of those calculations.

He was witty and extremely friendly with all of us. He knew how to approach everyone with advice, and expressed his opinion in a straightforward manner or even with a joke. When I showed him a draft of the long introduction of my Ph.D. Thesis, after having already read the remaining chapters, he quickly read the first three or four pages, looked at me smiling and asked me how I managed to get married. He saw my enormous surprise. He said that if I was so slow and introductory approaching women I would never have had even a girlfriend. He said that both in life and in mathematical writing one must be objective, that one must write in a succinct, clear, modest but convincing form about the importance of the topic or the problem and about his own contribution.

In recent years we used to talk, make jokes and laugh a lot during our rare meetings. Though he had more obligations as Dean of the Mathematics Department, as Editor-in-Chief of the East Journal on Approximations and many others, I never saw him so relaxed and laughing as recently. I was rather surprised when I managed to take some pictures of him. He didn't have a personal homepage at the time. I convinced him to take a picture in order to create his web site. In fact I

needed a nice photo to put in the volume dedicated to his 60th birthday we were preparing as a surprise. It was not easy to take the picture because he insisted that the problem of generalizing Markov's inequality for any order of the derivative and any L_p norm that he had written on the blackboard was more important than his face. The last time we could talk for a long time was a year ago during the traditional meeting on optimization and approximation in the Caribbean, held on the beautiful Colombian island of San Andrés. One day we had a long walk on the beach talking about Bulgaria, Brazil, about my students, about maths and even drawing lemniscates of polynomials in the sand. Then we sat in a small canteen on the street in front of the sea and continued talking, having juice and we took another picture. I sent him that photo only nine months later when we exchanged Christmas wishes. He adored the picture and replied that everyone would be jealous of us because we were happily having fresh tropical fruit juice on a little beautiful pirate island. He was like that, sometimes serious and rigorous, but most of the time friendly, jolly, laughing happily.

I'll miss you, Teacher!

Vladimir Tikhomirov (Moscow State University). The loss is bitter and grievous for each of us, for his country, for our journal, and for our branch of mathematics.

Borislav was very close to the Moscow school of approximation theory. He undertook training with Prof. N. S. Bakhvalov. Nikolay Sergeevich and I were opponents of Sergey Smolyak's Master's Thesis in 1965 where an essentially new direction in analysis, which is now called Optimal Recovery, was started (it was inspired by ideas of Andrey N. Kolmogorov who initiated an amazing number of new ideas at that time). Bakhvalov very much appreciated this new direction. He published an important result from Smolyak's thesis which was later called Smolyak's Lemma and which formed the basis for the new school of Optimal Recovery. Bakhvalov's pupils were Borislav Bojanov and Konstantin Osipenko. They started the development of two main branches of Optimal Recovery (similar to the research on entropy and widths of smooth and analytic functions carried out during the same years). Bojanov focused on recovery of functions with finite smoothness, while Osipenko concentrated on analytic functions.

During the 1960–1970s we were in close contact with Borislav, and I was a witness to his rapid advancement. In 1973 Borislav wrote a Ph.D. Thesis and I was supposed to be his opponent. I had to get a visa, but soon I became aware that one was not likely to be issued. I still recall the reason why my visa application was rejected with a smile: I was told that since they cater for me in Bulgaria, my evaluation would not be impartial!

Nonetheless, the thesis was duly defended and Borislav continued to advance in both science and public activity. He traveled around the world, making friends everywhere, drawing information from communication and sharing his knowledge generously. Our meetings were invariably very warm. Borislav and I also wrote a joint article for "Serdica".

Borislav's contribution to science is apparent, and his effort to unite us was great. He started publishing a journal in approximation theory during a difficult historical period for Eastern Europe and, at some point in time, he even published it at his own expense. Borislav organized many important conferences on approximation theory and he enjoyed visiting different countries to participate in various workshops. Two years ago he took part in a conference in Moscow which was organized by my pupils. We agreed that in the autumn I would go to Bulgaria, which I had not visited for a long time (Bulgaria was the first foreign country which I visited, and for a long time it was on the top of my list of trips abroad, but then there was a long break). Apart from visiting Bulgaria again, I was planning to discuss the past, the present and the future of our

branch of science with Borislav. Borislav and I had different approaches to some aspects of the theory.

I did not see Borislav last year, and I was planning to see him in Sofia this autumn. Alas, this is not to happen.

Nira Dyn (Tel-Aviv University). I met Borislav at the Varna Conference in 1981. We were both interested in Gaussian-type quadratures, and indeed later wrote a joint paper (also with Dietrich Braess) on generalized Gaussian quadratures, which was published in JAT in 1986.

After the conference in Varna, Borislav invited Allan Pinkus and me to visit his home town, Veliko Tarnovo, which was the medieval capital of Bulgaria. We took the train to get there, and we had a fantastic guided tour of the old part of the city, with Borislav as the guide. In this tour we learned a lot about the history of Bulgaria. I remember this day in Veliko Tarnovo as very interesting and very special.

After the tour Borislav went to see his mother for a short time, and asked Allan and me to wait for him. At those times it was very risky to invite Israelis to one's home, and Borislav did not want to make problems for his mother. I am sure that to be together with Israelis was also risky, and I appreciate very much the courage of Borislav on top of his hospitality. When he came from his mother he brought us presents from her — Bulgarian doilies, which I still keep.

Vitalii Arestov (Ural State University, Ekaterinburg). Borislav Bojanov has obtained a number of significant results concerning algebraic and trigonometric polynomials. In 1939, Erdős proved that the function $\cos nt$ and its translates (and only these polynomials) have a maximal arc-length on the period 2π among trigonometric polynomials of order $n \geq 1$ with a uniform norm equal to 1. He also conjectured that the Chebyshev polynomial $T_n(x) = \cos n \arccos x$ has maximal arc-length in $[-1, 1]$ among algebraic polynomials of order $n \geq 1$ with uniform norm $\|f_n\|_C = \max\{|f_n(t)| : t \in [-1, 1]\}$ bounded by 1 in $[-1, 1]$. It was Bojanov who proved this conjecture of Erdős after more than 40 years; see [49]. To be more exact, he proved that in the set $\mathcal{P}_n$ of algebraic polynomials of order $n \geq 1$, the following inequality holds:

$$\int_{-1}^{1} \sqrt{1 + |f_n'(t)|^2}\, \mathrm{d}t \leq \int_{-1}^{1} \sqrt{1 + \|f_n\|_C |T_n'(t)|^2}\, \mathrm{d}t, \quad f_n \in \mathcal{P}_n, \tag{1}$$

and an equality is attained if and only if $f_n = M\, T_n$, where M is an arbitrary real number.

In fact, Bojanov obtained a considerably more general result; see [50, 48, 54]. Namely, he proved that if a function φ is increasing and convex on the half-line $[0, \infty)$ then the following inequality holds:

$$\int_{-1}^{1} \varphi(|f_n'(t)|)\, \mathrm{d}t \leq \int_{-1}^{1} \varphi\left(\|f_n\|_C |T_n'(t)|\right)\, \mathrm{d}t, \quad f_n \in \mathcal{P}_n. \tag{2}$$

Moreover, only the polynomials $f_n = M\, T_n$ are extremal in (2). Inequality (1) is a specific case of (2) for the function $\varphi(u) = \sqrt{1 + u^2}$. For the function $\varphi(u) = u^p$, $p \geq 1$, inequality (2) turns into the inequality

$$\|f_n'\|_{L_p} \leq \|T_n'\|_{L_p}\, \|f_n\|_C, \quad f_n \in \mathcal{P}_n, \tag{3}$$

with the L_p-norms of the polynomials. Thus (3) is an extension of the known Markov's inequality.

Studying quadrature formulae and interpolation processes with multiple nodes, Bojanov arrived at the necessity of studying generalized Chebyshev polynomials. Let $\nu = (\nu_1, \ldots, \nu_n)$

be a set of n natural numbers. For n points $-1 \le x_1 \le \cdots \le x_n \le 1$ from $[-1, 1]$, we construct the algebraic polynomial

$$f_N(x) = \prod_{j=1}^{n} (x - x_j)^{\nu_j} \tag{4}$$

of order $N = \nu_1 + \cdots + \nu_n$. The problem is to find a polynomial of the form (4) (i.e., to select its multiple zeros $\{x_j\}$) with a smallest uniform norm in $[-1, 1]$. The existence of such a polynomial and its properties was discussed in a few works by Bojanov, beginning with [39]. Bojanov and Rahman, see [81], extended a number of sharp classical inequalities to polynomials (4). In particular, they proved an analogue of inequality (3) for derivatives of any order k, $1 \le k \le N$.

As a rule, problems for algebraic and trigonometric polynomials are beautiful and their statements are well understandable. Moreover, a plausible conjecture about an extremal polynomial solving the problem can often be suggested. However, for the most part, these problems are rather difficult. To solve them one needs a new idea and a new method, which in turn usually allows one to solve a number of other problems. Bojanov's method is based on the result that a number of functionals, in particular, the left-hand sides of inequality (2) and so (1) and (3), are monotone functions of the vectors $\{|f_n(\xi_j)|\}$ of absolute values of the local extrema of a polynomial f_n in $[-1, 1]$ (including the endpoints ± 1). This idea can be found in many of his works. In a complete form it is presented in the joint paper by Bojanov and Rahman, [81].

Using this idea, Borislav Bojanov, his colleagues and disciples obtained solutions to a number of problems in function theory. In particular, Bojanov and Naidenov, see [100], applied it to the investigation of the sharp Landau–Kolmogorov inequality on a finite interval. In addition, they proved the validity of another conjecture by Erdős from 1939, namely that only an appropriate translation of $\cos nx$ has maximal arc-length over any interval $[a, b] \subset [0, 2\pi]$ among all trigonometric polynomial of order n with a uniform norm bounded by 1 on the period.

Along with Markov-type inequalities, Bojanov also obtained considerable results for Shur-type and Turán-type inequalities; see [78]. He published two detailed surveys ([78] and [113]) on sharp inequalities for polynomials and splines.

Works by Bojanov contain beautiful original results, and they are written very skillfully. Borislav Bojanov has handed down to his disciples and to the entire mathematical community a number of significant ideas which will certainly allow one to obtain new knowledge regarding difficult extremal problems in function theory.

Hakop Hakopian (Yerevan State University). It was during the International Conference on Approximation Theory in Kiev in 1983. Several Armenian participants, of which I was the youngest, told me that a well-known Bulgarian mathematician was praising my results in an unusual way. It must be noted that my compatriots were very surprised, because they knew me merely as a beginning Ph.D. student. At the same time they were very pleased to the extent that they still tell me about this today. I became acquainted with Borislav at the banquet at the end of the conference. The next year, Borislav proposed to my adviser – Professor Zbigniew Ciesielski – to immediately present my Ph.D. Thesis to the Habilitation degree. Professor Ciesielski liked the idea. I was very proud, though would have preferred to proceed in the usual way. Later we became close friends and I received his help and encouragement many more times. For me it was very easy to ask Borislav for any kind of help and I was confident that he was there. He was a very kind, sincere, and openhearted person. One could always find a smile on his face. I am very grateful to fate that enabled me to meet and collaborate with such a man.

Borislav was extremely modest person. One could not at all deduce from his behavior that he was a world-class mathematician who also created an internationally recognized mathematical school. He was a great patriot of Bulgaria. I remember that on one occasion, in Soviet Union times, he complained about one of our colleagues (from an independent country) who had claimed that there was no need for further independent states in the world. It is worth noting that we then frequently communicated in Polish (both of us studied in Poland for a period). This was, I think, a sign of solidarity with the liberation struggle of the Polish people. When I wrote him that I had applied to finish my teaching abroad he replied: I think your decision to come back to Armenia is wise. Life is hard in our countries but it is much more interesting and healthy than abroad.

I think that he was not financially secure. There was a time he was thinking of joining me to teach in Saudi Arabia for a short period. I also know that recently on one occasion he chose to travel by bus rather than airplane to travel from Sofia to Athens. He had to build his summer house and do the work there by himself. And as far as I know he did not do much there at all.

Once he was to visit Saudi Arabia in 2005 as an opponent of a Ph.D. student of mine. He had very complicated transit flights. I went to Riyadh Airport to meet him but he was not among the passengers. I was frightened and did not know what to do. But anyway I did not contact his family about this. Fortunately, the next night I received information from the airport that he was arriving. I met him and told that I had been thinking about phoning Rositza – his wife – and telling her that he had not arrived. He interrupted immediately and exclaimed: No, no, no, you should not have done this – as if trying to prevent me doing a thing in the past – yesterday! One could feel at once how much he cared for and loved his family.

Borislav could make observations which were a kind of advice. Once he told me that I should have participated in a meeting, very important for me, which I missed. I brought forth several reasons, each of which according to me made my presence really impossible. He did not say anything. But the next day he repeated his observation.

He was good company and liked to tell curious things. I had the pleasure to have him in Armenia during the International Conference on Harmonic Analysis and Approximations, III, in 2005. I was taking him from the airport to the conference site. At a cross-road I told him that we would turn to the left, while the straight way went to Lake Sevan. What? Is Sevan a lake? He became surprised. Do you know that Noah's Ark is there? Now I was surprised: What, it is not on the Ararat but in Sevan? Then he told me that recently he knew that once Vanga, the famous Bulgarian clairvoyant, was asked whether the Bible story about Noah is true? The answer was: Yes, it is true and the Ark is in Sevan now.

Also I know that Borislav would not listen quietly even to his closest friends saying anything bad about any far distant person whenever he found it not fair. It seems to me that the truth was more important to him than personal, political or practical considerations. At the same time he took other people's pain as his own. He would do everything to help his friends. In Armenia, on the way back to the airport, I told Borislav that I was in a really bad situation – I had been refused admittance to a teaching position in the department where I worked before my leave from Armenia. He was surprised and said: Perhaps X can help you; he has a serious position in the Department, hasn't he? My answer was: X finds it normal what happened. Borislav was silent for some time. Then he told me: Take your family and come to Bulgaria.

Borislav was full of energy and kindness. His death was so unexpected. I am very sorry that I was not able to take part in the last farewell of my best friend. But I know that Borislav will be among those few meeting me there, with his kind smile.

Ronald DeVore (University of South Carolina, Columbia and Texas A & M, College Station). I first met Borislav in the late 1970s at one of the approximation theory conferences which were held every three years or so in Varna. Early on these were important meetings to hook up with what was happening in Eastern European mathematics — the birthplace of what we call Constructive Function Theory. Borislav seemed like a nice fellow but I really did not know him that well until he visited Columbia sometime in the mid-1980s. During our mathematical discussions during this visit, he honed in rather quickly on some delicate mathematical arguments in L_1 approximation and showed me some very useful tricks of the trade. It was then I knew of his mathematical gifts. I respected his talents from that point on. I also benefited by querying him on mathematical problems through the remaining years. I gradually began also to see his sympathetic nature and his concern for students. He sent me two great students to South Carolina in Guergana Petrova and Bojan Popov. We also established a more firm connection between Bulgaria and South Carolina through exchange programs. Through the years I benefited from visits of scores of Bulgarian scientist to South Carolina. South Carolina was even fortunate to retain some (Pencho Petrushev, Peter Binev and Ognian Trifonov) of these talents. Other people, who know the details, can report on what Borislav did for mathematics inside Bulgaria. I heard many times that he was the driving force for keeping mathematics and approximation theory alive through the bad times (post perestroika).

Borislav was a very kind man, a gentleman in every respect. The last time I visited Bulgaria for a conference in Varna a few years back it was at the invitation of Borislav for a conference they were holding to honor Professor Sendov's birthday. Borislav handled every detail of my visit in what seemed like an effortless manner. On the day of my departure, I had an early flight and had to leave the hotel at some ungodly morning hour. I asked Borislav to help order me a taxi which he agreed to do. When I came down to the lobby at 5 AM the next day, there was Borislav ready to escort me in the taxi to the airport. I miss Borislav and reflect on him often.

Szilárd Gy. Révész (Hungarian Academy of Sciences, Budapest). I have known Borislav somewhat closely only relatively recently, say for six years. Nevertheless, I quickly came to the feeling that I had a very good friend, perhaps my best friend among all my foreign colleagues. I do have some, so that definitely means something. It took me a while to understand, and it is still haunting me, that I will never again meet him in Bulgaria. For me he belonged to the essential characteristics of Bulgaria, as a visit there could not be imagined without meeting him. But now I am writing some recollections in order to at least record, perhaps save, some of my feelings and impressions as otherwise they will be lost.

He liked to recall stories. One of his favorites was the first encounter he had with Stechkin. Already at the stage of a young Ph.D. student he had the integrity not to strive to please others — not even Stechkin — but politely express his stand. Which was not bad, not even to Stechkin, who gradually accepted, even respected the young man with strong integrity. The story was about Stechkin attacking him at their very first meeting, for not quoting some Russian literature — was it something from Stechkin himself? I do not remember exactly — in relation to some facts concerning polynomials. And Borislav's answer: "If I had to quote something, I would have quoted Obreshkov's book." Later in his professional life Borislav did a lot to translate and publish some really magnificent works of Obreshkov and others in English. It was heart-warming to see how the Bulgarian Academy kept publishing such pearls even during the worst economic situation. Borislav's respect for his Bulgarian mathematical predecessors and professors did not exhaust itself in declarations. He worked persistently to popularize the mathematical heritage of them — of Bulgaria.

Borislav was a romantic person. I did not know him sufficiently closely as to judge about his personal life, although it was clear to me that he definitely very much liked his family, and was very much a family-centered person. However, the reason I consider him a romantic person comes from his stories. He frequently told stories which also reflected his views about life, and I think that not only were the stories somewhat romantic, but also Borislav was a romantic person.

I liked very much the one about one of his colleagues, Bozhidar Sendov, and an old man hiking up the Vitosha Mountain. The story came from the colleague and it was about his experience when, on going up the Vitosha Mountain, an old chap always got ahead of him. Bozhidar Sendov, a rather sporty young man, got annoyed seeing that an old guy was always ahead of him and that he could not pass him. So he started to accelerate. However, the old man simply accelerated in the same way, and still kept ahead of him. This went on for hours until they arrived at the peak. There was a small drinks bar with wooden chairs and tables serving beer, tea, and even some food. He sat down exhausted. And then a hand kindly touched his shoulder. It was the old man. "Don't be angry with me young man," he started, "I'll tell you how it is. I go up the hill every day, for decades, all through my life. The mountain has helped me heal from the loss of my beloved wife and the painful memories of the circumstances of how she died. I come up here every day since. That is why I am so trained and that is why you could not catch me. Sorry for that. . . ." And then the old man told his story. What is nice is that Borislav's colleague passed this on, and Borislav too, as a serious lesson from life. It is characteristic of a man, of a community, of a culture, whose stories live on and are passed down, even to foreigners like myself.

This story was told to me on the occasion of a hiking trip to the Vitosha. All of us knew that Borislav loved the mountains, in particular his Balkan mountains, the Vitosha, the Rila mountains .... And he loved hiking, and sports, in general. At the age of 60 he regularly ran between the University and the Institute of Mathematics and Informatics of the Bulgarian Academy. All too often he had no time to spend on going from one place to the other, but sports made this a reasonable utilization of time.

It was the fall of 2005 when I spent some more time in Sofia under an exchange agreement between our Academies. Borislav's office was a really dingy place in an outdated building. The wind flew through the iron window frames and it was rather cold. Furniture looked as if it had remained there from a previous age. Nevertheless, once a week a cheerful company of his colleagues and students came together in this office to have tea together. Half of Borislav's cupboard was packed with tea, various jugs and glasses; stuff for tea. The room became fully crowded, so when I tried to take a photo of the nice company I finally decided to stand up on a desk in order to have a perspective where most people could be seen. Irina, the other dedicated photographer, took the opportunity and caught me standing on a desk in Borislav's office among a considerable crowd.

Borislav was not only romantic, he was also optimistic. As Erdős, he also liked to express his views in mathematical terms. He had been through difficult times in Bulgaria. But ever since I knew him he liked to point out that for the people's psychological evaluation of the situation, not the actual state of affairs really mattered. The derivative is what is important and the derivative is positive, he said. Sometimes when we did not have time to discuss things in detail, as we were just meeting for a short encounter, it was sufficient to ask him what the situation was like and whether the derivative was positive. It always was. I am not sure if this unchangeable positivity is really characteristic of Bulgaria — perhaps more characteristic of Borislav.

Borislav was a responsible and dedicated organizer. We arranged two consecutive exchange agreement projects between our Academies. Under this framework he several times organized workshops. The one in Sozopol was a particularly nice one, quite memorable to my young

colleagues who had the good luck to take part. The opening session was in the open-air beach bar, with a bed sheet put up on the wall for projectors, and a small whiteboard for writing. Borislav gave the opening address. It was something like this, freely quoted. "Well, I always had a few dreams in my life which I would have liked to reach. One of those was to organize a conference without lectures. This time I came quite close to achieving it. But for some administrative reasons, some of our young colleagues from Hungary need to deliver a talk. Let us therefore have a few lectures before going to the beach."

Otherwise he had few dreams. Also, in a nice way he used to point out that only a part of one's dreams could be attained. At least to me that was the lesson from his frequent quoting of his own dreams and giving an account of what he had actually accomplished. He said he would have liked to have had six children, and he has three. He would have liked to run, at least once, a marathon, but actually at his best he could have run a half-marathon. He never did mention it that way, but I am sure he could have added to this list of half-finished accomplishments of dreams his term as the Dean of the Faculty of Mathematics and Informatics at the St. Climent Ohridsky University of Sofia. He was a very successful, active, courageous and inventive dean who pushed the faculty from a standstill to activity and development. Considering the financial circumstances and disorder it was a rather bold action to start the renewal and upgrading of the building of the faculty, which he managed to accomplish against all odds. A less dramatic, but psychologically perhaps even more elevating story is the cleaning of the neglected wooden corner of the land near where the building stands. Nowadays this park is a pearl within the university, where garden parties and conference receptions are held. Nevertheless, when he wanted to continue his activity he could not be re-elected under some regulation discriminating on the basis of age. That was perhaps the first serious situation when he was rejected and prevented from doing his best for his beloved scientific community. All of us around him felt that this bothered him deeply.

He liked his homeland and expressed the view that he would stay at home, in Bulgaria, under almost any circumstances. Since I am a similar character, we could get along rather easily. Naturally, our relationship was not a fully symmetric one: I more often felt his support than I could do anything for him.

Borislav had a very nice company of colleagues and students around him. I know that they miss him very much. So do I. But I still hope that we will maintain our professional and personal contacts even after Borislav's passing away. I think he would have wanted us to do so.

Kamen Ivanov (Bulgarian Academy of Sciences, Sofia). I first met Borislav Bojanov in the late 1970s at the "Mathematical Modeling" section of the United Center for Mathematics and Mechanics of the Bulgarian Academy of Sciences. At that time we had frequent contacts as members of the Bulgarian group working on approximation theory. From these first meetings Borislav strongly impressed me by his devotion to mathematics and his dissatisfaction when distracted from his work. He participated in the discussion of many mathematical results and open problems with passion and enthusiasm, I would say with an "internal burning".

Borislav Bojanov is descended from a family of priests, which was a disadvantage in our communist society until the end of the 1980s. As a consequence of this he took the line of keeping a low profile and not accepting any leadership positions in the mathematical community. This mindset was strong even in the late 1990s, almost ten years after the fall of communism. I remember very well the long conversations we had at that time when I managed to convince him to accept the position of chairman of the Mathematical sub-commission of the Bulgarian High Attestation Commission.

Knowing his attitude, I was pleasantly surprised when in 2003 Borislav Bojanov accepted the position of Dean of the Department of Mathematics and Informatics at Sofia University. In my judgment, his term of office was one of the most fruitful periods of the department's history. Borislav initiated and accomplished a program for renovation and extension of the department building. He put a lot of effort into raising money from a variety of state and private sources — a complicated task in a difficult economic period. Simultaneously with the building program, Borislav Bojanov organized a foundation for supporting talented mathematical students.

One of my strongest recollections of Borislav Bojanov is from 1983, after he proved the Erdős conjecture on the "longest" algebraic polynomial. He was both very happy and strongly inspired by this achievement. At a meeting he shared with us that now he could die in peace even if he would do nothing more in mathematics. It turned out that Borislav had an additional 25 years of life full of productive work and beautiful mathematical results.

Borislav Bojanov put a lot of effort into supervising his Ph.D. students. He taught them to value the beauty in mathematics as he did. He had a strong religious belief in the good in everyone. In this sense he was the "eternal optimist". I will remember his friendly smile, his endless energy and his pride in being a Bulgarian.

Gradimir Milovanovic (Megatrend University, Beograd). I knew Borislav Bojanov for a long time, for more than 25 years. Our first contacts were in the 1980s at several conferences (Sofia, Varna, Niš), as well as at local meetings between mathematicians from Sofia and Niš. In 1987 Borislav joined the Editorial Board of the journal "Facta Universitatis, Series: Mathematics and Informatics", which was started a year earlier at the University of Niš.

Professor Borislav Bojanov was an extraordinary person, a great scientist and a very modest person. He was a very witty man, ready with a joke at any time. Among the many jokes I remember, for instance, was his question: *"What is the difference between a camel and a man"*, with the answer: *"A camel is able to work constantly for seven days without drinking at all, while a man is able to drink constantly for seven days without working at all!"* His jokes were short, with an intelligent humor, and were often filled with mathematical content.

In that period I worked on my book *Topics on Polynomials: Extremal Problems, Inequalities, Zeros*, published in 1994 by World Scientific. Borislav always showed interest in the progress of that comprehensive project and encouraged me to successfully finish that job. Polynomials occupied an important place in his very impressive scientific work. His first published paper was devoted to estimates for the zeros of algebraic polynomials, see [1], and was based on his Master's Thesis from the University of Wroclaw in 1968, under the supervision of Professor Stefan Paszkowski. He very much liked the subject of polynomials, polynomial inequalities and extremal problems with polynomials, producing very nice and important contributions. This area has become a challenge also for his students. Several problems concerning polynomials with multiple zeros were the subject of Borislav's interests.

In connection with the classical interpolation process on the basis of the nodes $\{t_\nu\}_{\nu=1}^n$, $a \le t_1 < \cdots < t_n \le b$, with the fixed system of multiplicities $\mathbf{k} = \{k_\nu\}_{\nu=1}^n$, respectively, Bojanov, see [39], stated the extremal problem of determining

$$\inf_{t_1<\cdots<t_n} \|(t-t_1)^{k_1}\cdots(t-t_n)^{k_n}\|_\infty$$

and he proved the existence and uniqueness of the extremal nodes $\{t_\nu^*\}_{\nu=1}^n$. Evidently, the solution in the simple case, $b = -a = 1$, $k_1 = \cdots = k_n = 1$, leads to the Chebyshev polynomials of the first kind. The generalized Chebyshev polynomials $T_m(t;\mathbf{k}) = (t-t_1^*)^{k_1}\cdots(t-t_n^*)^{k_n}$, $(m = k_1+\cdots+k_n)$ preserve some properties of the classical Chebyshev polynomials (see [54]).

Inspired by results of the famous Bulgarian mathematician Chakalov from 1954 on the weighted L^2-extremal problem

$$\inf_{t_1<\cdots<t_n} \int_a^b w(t)|(t-t_1)^{k_1}\cdots(t-t_n)^{k_n}|^2\,\mathrm{d}t,$$

which is connected with quadratures of the highest algebraic degree of precision, Borislav proved the existence and uniqueness of extremal points in the corresponding problem in L^r-norm ($1 < r < \infty$) and L^1-norm; see [35, 52].

Another nice result of Bojanov, see [50], is an extension of the classical Markov inequality from 1889, proving that the Chebyshev polynomial T_n preserves its extremal property in the corresponding L^r-problem. Namely, for every polynomial $P \in \mathcal{P}_n$ ($\mathcal{P}_n$ is the set of polynomials of degree at most n) and $r \in [1, \infty)$, he proved the inequality

$$\|P'\|_r \le \|T_n'\|_r \|P\|_\infty,$$

with equality only for $P(t) = cT_n(t)$, where c is an arbitrary constant. Moreover Bojanov, see [48], proved that for every increasing convex function $\varphi(x)$, for $x > 0$, the quantity

$$\sup_{P\in\mathcal{G}_n} \int_{-1}^{1} \varphi(|P'(t)|)\,\mathrm{d}t \quad \left(\mathcal{G}_n = \left\{P \in \mathcal{P}_n : \|P\|_\infty \le 1\right\}\right)$$

is attained only for $P(t) = \pm T_n(t)$. For $\varphi(x) = \sqrt{1+x^2}$ this result provides the solution of a conjecture on the longest polynomial stated by Erdős in 1939. Namely, among all polynomials of the class $\mathcal{G}_n$, only the polynomials $\pm T_n(t)$ have maximal graph length in $[-1, 1]$. This result was proved by Bojanov in [49] more than 40 years after it was stated by Erdős.

These beautiful ideas were extended and refined by Bojanov to obtain significant Markov- and Turán-type polynomial inequalities for algebraic polynomials with multiple zeros, for oscillating trigonometric polynomials and perfect splines; see [86, 98, 110].

Recently, Bojanov gave a plenary lecture at the conference "Approximation and Computation" (Niš, 2008), see [131], and it will be published very soon by Springer. It is a survey on some particular polynomial problems that are related to complex analogues of Rolle's theorem or to the Bernstein majorization theorem that implies the well-known estimate for the derivative of a complex polynomial on the disk. The main topic, however, is Sendov's conjecture about the critical points of algebraic polynomials. Despite the numerous attempts to verify the conjecture it is not yet settled and remains one of the most challenging problems in the analytic theory of polynomials. Bojanov also discusses the mean value conjecture of Smale and points out certain relations between these two famous open problems. Finally, he formulates a conjecture that seems to be a natural complex analogue of Rolle's theorem and contains as a particular case Smale's conjecture.

His scientific work was very fruitful, especially in the field of optimal interpolatory and quadrature processes, as well as optimal recovery of functions and functionals. In addition he had substantial results on non-standard quadratures of Gaussian type in one and several dimensions. These quadratures are based on non-standard information data different from the standard one consisting of function values at a finite number of points.

It was a great pleasure to cooperate with such a great scientist and remarkable person as Borislav Bojanov. In the last period we had an intensive cooperation, in particular, on the SCOPES Joint Research Project *New Methods for Quadrature* (2006–2008) (jointly with

Professor Walter Gander, ETH, Zürich), supported by the Swiss National Scientific Foundation. Our cooperation and friendship will remain forever in my memory.

List of Doctoral Students of B.D. Bojanov

1. Georgi Grozev (1987)
2. Geno Nikolov (1990)
3. Rumen Uluchev (1990)
4. Dimitar Dimitrov (1992)
5. Hrisina Draganova (1998)
6. Lozko Milev (2000)
7. Natasha Dicheva (2001)
8. Petar Petrov (2001)
9. Nikola Naidenov (2003)
10. Irina Georgieva (2005)

List of Publications of Borislav D. Bojanov

Papers

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6. B. Sendov, B. D. Bojanov, On a property of a class of linear positive operators, *Annuaire Univ. Sofia, Fac. Math.* **64** (1969/70), 115–117 (1971). MR0313684 (47 #2238)
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