intervalla HI, IK, KL, LM, &c. unitates esse, & dic $AH=a$, $-HS=p$, $\frac{1}{2}p$ in $-IS=q$, $\frac{1}{3}q$ in $+SK=r$, $\frac{1}{4}r$ in $+SL=s$, $\frac{1}{5}s$ in $+SM=t$; pergendo videlicet ad usque penultimum perpendiculum ME, & præponendo signa negativa terminis HS, IS, &c. qui jacent

jacent ad partes puncti S versus A, & signa affirmativa terminis SK, SL, &c. qui jacent ad alteras partes puncti S. Et signis probe observatis, erit $RS=a+bp+cq+dr+es+ft$, &c. LIBER TERTIUS.

Cas. 2. Quod si punctorum H, I, K, L, &c. inæqualia sint intervalla HI, IK, &c. collige perpendiculorum AH, BI, CK, &c. differentias primas per intervalla perpendiculorum divisas b, $2b$, $3b$, $4b$, $5b$; secundas per intervalla bina divisas c, $2c$, $3c$, $4c$, &c. tertias per intervalla terna divisas d, $2d$, $3d$, &c. quartas per intervalla quaterna divisas e, $2e$, &c. & sic deinceps; id est, ita ut sit $b=\frac{AH-BI}{HI}$, $2b=\frac{BI-CK}{IK}$, $3b=\frac{CK-DL}{KL}$, &c. dein $c=\frac{b-2b}{HK}$, $2c=\frac{2b-3b}{IL}$, $3c=\frac{3b-4b}{KM}$, &c. postea $d=\frac{c-2c}{HL}$, $2d=\frac{2c-3c}{IM}$, &c. Inventis differentiis, dic $AH=a$, $-HS=p$, p in $-IS=q$, q in $+SK=r$, r in $+SL=s$, s in $+SM=t$; pergendo scilicet ad usque perpendiculum penultimum ME, & erit ordinatim applicata $RS=a+bp+cq+dr+es+ft$, &c.

Corol. Hinc areæ curvarum omnium inveniri possunt quamproxime. Nam si curvæ cujusvis quadrandæ inveniantur puncta aliquot, & parabola per eadem duci intelligatur: erit area parabolæ hujus eadem quamproxime cum area curvæ illius quadrandæ. Potest autem parabola per methodos notissimas semper quadrari Geometrice.

7] divisa: divisas M E_1 E_2 *and Corrigenda to* E_3 (*corrected by hand in the copy of* E_3 *reproduced; see note on p. 38 of the present edition*)

13] s [*twice*]: S M E_1 *but* E_1a = E_3

CASE 2. But if HI, IK, &c., the intervals of the points H, I, K, L, &c., are unequal, take b, $2b$, $3b$, $4b$, $5b$, &c., the first differences of the perpendiculars AH, BI, CK, &c., divided by the intervals between those perpendiculars; c, $2c$, $3c$, $4c$, &c., their second differences, divided by the intervals between every two; d, $2d$, $3d$, &c., their third differences, divided by the intervals between every three; e, $2e$, &c., their fourth differences, divided by the intervals between every four; and so forth; that is, in such manner, that b may be $=\frac{AH-BI}{HI}$, $2b=\frac{BI-CK}{IK}$, $3b=\frac{CK-DL}{KL}$, &c., then $c=\frac{b-2b}{HK}$, $2c=\frac{2b-3b}{IL}$, $3c=\frac{3b-4b}{KM}$, &c., then $d=\frac{c-2c}{HL}$, $2d=\frac{2c-3c}{IM}$, &c. And those differences being found, let AH be $=a$, $-$HS $=p$, p into $-$IS $=q$, q into $+$SK $=r$, r into $+$SL $=s$, s into $+$SM $=t$; proceeding in this manner to ME, the last perpendicular but one; and the ordinate RS will be $=a+bp+cq+dr+es+ft+$&c.

COR. Hence the areas of all curves may be nearly found; for if some number of points of the curve to be squared are found, and a parabola be supposed to be drawn through those points, the area of this parabola will be nearly the same with the area of the curvilinear figure proposed to be squared: but the parabola can be always squared geometrically by methods generally known.

LEMMA VI

Certain observed places of a comet being given, to find the place of the same at any intermediate given time.

Let HI, IK, KL, LM (in the preceding fig.) represent the times between the observations; HA, IB, KC, LD, ME, five observed longitudes of the comet; and HS the given time between the first observation and the longitude required. Then if a regular curve ABCDE is supposed to be drawn through the points A, B, C, D, E, and the ordinate RS is found out by the preceding Lemma, RS will be the longitude required.

By the same method, from five observed latitudes, we may find the latitude at a given time.

If the differences of the observed longitudes are small, let us say 4 or 5 degrees, then three or four observations will be sufficient to find a new longitude and latitude; but if the differences are greater, as of 10 or 20 degrees, five observations ought to be used.