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In memoriam

In Memoriam: Mizan Rahman

Mizan Rahman was born on September 16, 1932 in East Bengal, India. Mizan studied at the University of Dhaka where he obtained his B.Sc. degree in Mathematics and Physics in 1953 and his M.Sc. in Applied Mathematics in 1954. He received a B.A. in Mathematics from Cambridge University in 1958, and an M.A. in Mathematics from Cambridge University in 1963. He was a senior lecturer at the University of Dhaka from 1958 to 1962. Mizan decided to go abroad for his Ph.D. He went to the University of New Brunswick in Canada in 1962 and received his Ph.D. in 1965 with a thesis on kinetic theory of plasma using singular integral equations techniques. The traditional British applied mathematics curriculum included many subjects which nowadays will be considered theoretical physics. This was the case in many former British colonies including India.

After obtaining his Ph.D., Mizan became an assistant professor of Mathematics at Carleton University. He rose through the ranks of associate professor, full professor and upon his retirement he became professor emeritus at Carleton University. He was designated as a Distinguished Research Professor after his retirement. He unexpectedly passed away in Ottawa on January 5, 2015 at the age of 82.

The book *Theory and Applications of Special Functions* [8] from 2005 is dedicated to Mizan Rahman, and originated from a special session at the American Mathematical Society Annual Meeting in Baltimore, Maryland, January 2003, organized by Mourad Ismail. In particular, the book contains many contributions on special functions, the main research subject of Mizan Rahman after his switch from theoretical physics to mathematics. In the introductory paper several recollections of Mizan and his work can be read. In particular, Michael Hoare's recollection of this switch to mathematics can be found as well as some reminiscences of his sons, Babu and Raja. Also included in [8] are two papers that Mizan wrote jointly with George Gasper. The first paper, 109 in the list below, is on multivariable biorthogonal polynomials and the second one, 110 in the list below, on multivariable Askey–Wilson polynomials. The first paper in [8] contains a list of Mizan Rahman's publications up to 2002, see 2b) on [8, p. xx], of exactly 100 papers and we have updated this list. More information on Mizan, such as interviews and pictures, can be found at the webpage mentioned in the footnote to [6].

One of Mizan's most influential post-2002 publications is the second edition of *Basic Hypergeometric Series*, which he coauthored with George Gasper. The subject of basic hypergeometric series goes back to the 18th century and to mathematicians such as Euler and Heine. The subject has seen an enormous boost over the last three decades, and it has been seen to be intimately related to various subjects, e.g. combinatorics, partition theory, number theory,

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Mizan in Hong Kong in 2010 (left) and at Yosemite in 2008 (right).

orthogonal polynomials and special functions, harmonic analysis on quantum groups, probability theory, integrable systems, mathematical physics, etc. For many of the scientists working in these fields, the book *Basic Hypergeometric Series* has become a very important and much appreciated source of information. Both the first and second editions are tremendously useful, and Mizan and George have done the scientific community a great service in writing this book.

According to George Gasper the book was written because Mizan was frustrated that he could not find the formulas he needed. From George's email [6].

> We started collaborating on our joint research about four years later after we met a second time in San Antonio at the January, 1980, AMS meeting. Since he was tired of having to repeatedly search papers for known formulas involving basic hypergeometric functions that were not contained in either Bailey's or Slater's books, Mizan suggested that we should write an up-to-date book on basic hypergeometric functions. After some hesitation to start such an ambitious project, I eventually agreed that we should write the book.

The first edition appeared in 1990, and was an immediate success. One of the great features of the book was the way it added structure to many, seemingly isolated results in basic hypergeometric series. The book appeared at the right time, since there was a flurry of research activities in mathematics and mathematical physics involving basic hypergeometric series, and in no time the book *Basic Hypergeometric Series* was THE book on the subject. Soon after its publication a Russian translation by Natig Atakishiyev and Sergei Suslov appeared in 1993.

In 2004 a second edition was published. The second edition is really an extension of the first edition, since Mizan and George added three chapters. These chapters are on generating functions for basic orthogonal polynomials, a q-extension of Appell's bivariate series and on elliptic hypergeometric series and integrals. The last subject was not yet available in 1990 when the first edition was published, since it started with the 1997 paper [4] by Frenkel and Turaev. In the last few years elliptic hypergeometric series and integrals have received a lot of attention, and many new results and extensions have been obtained. The introductory Chapter 11 in the second edition is a good introduction to the subject. Some of the material on generating functions was discovered after the publication of the first edition.

The first edition of 108 was not the first book on q-series. Parts of the theory were in Bailey's monograph [3] and in Slater [19]. Bailey's book was a Cambridge Tract, so it was very concise and had a brief introduction to q-series. Slater's book was more detailed and comprehensive but represented the state of the art as about 1960. George Andrews published his very influential CBMS lecture notes [1] in 1986 and demonstrated the breadth and wealth of the subject, but the notes were brief. Nathan Fine published [5] in 1988 where he covered parts of the theory using

his own approach. The book by Gasper and Rahman was very comprehensive and incorporated most of the major developments in q-series that took place in the 1980s. The second edition contained additional material as explained earlier and represented the state of the art when it appeared.

As stated before, Mizan's career started out in mathematical/theoretical physics. Moving to special functions only a little later, his main contributions are in special functions. Mizan had a special knack for the underlying structures in formulas based on special insights, supported by many years of experience. Especially, he was able to come up with proofs based on transformation and summation formulas of conjectural identities or of identities proved in another way. In this way, many mathematicians, including both authors, have benefited regularly from Mizan's insights and papers, since Mizan was always willing to help and share his ideas.

One highlight is Mizan's paper [9] with Mourad Ismail on the study of the associated Askey–Wilson polynomials. The paper is the first in a long collaboration between Mizan and Mourad, which ended with their last joint paper, published posthumously in 2015. In particular, the paper [9] describes the solutions to the Askey–Wilson q-difference eigenvalue equation explicitly in terms of very-well-poised ${}_8\varphi_7$-functions. Moreover, it describes the relations between the various solutions of this eigenvalue equation. These solutions turned out to be of fundamental importance in the study of Askey–Wilson functions [7,12,21] and the corresponding integral transforms and orthogonality relations at the turn of the millennium. In particular, all solutions have been determined including the solutions behaving well at infinity in a suitable sense. This leads to so-called c-function expansions, which are important in the study of the Askey–Wilson functions. These in turn are very important in physics [18] and currently multivariable analogues are being studied [20].

Another of his amazing results is contained in Mizan's paper [17], jointly with Arun Verma, presenting an addition and product formula for the continuous q-ultraspherical polynomials, which is a q-analogue of the addition formula for Gegenbauer (or ultraspherical) polynomials. The structure of the addition and product formula shows Mizan's great insight in identities for special functions, since the proof only uses results from basic hypergeometric series. It starts off with proving the product formula, and from there, noting that the obtained kernel is related to an Askey–Wilson weight function, the addition formula is obtained. Rahman and Verma show that the coefficients of this expansion factor as the product of two continuous q-ultraspherical polynomials, leading to the addition formula. Obtaining the addition formula from the product formula is a well-known technique, see e.g. [13,14] for the case of the addition formula for Jacobi polynomials. The classical addition formula for Gegenbauer polynomials follows from considerations using the differential equation or the group theoretic interpretation. Only much later [10] another proof for a special case of the Rahman–Verma addition formula using quantum groups was developed. Again, later Mizan proved the rather non-trivial identities needed to obtain the Rahman–Verma addition formula from the even more general addition type identity [11] obtained, see [11, Lemma 5.6]. This is yet another example of Mizan helping out another researcher. The paper [17] is one of four(!) papers by Mizan published in SIAM Journal on Mathematical Analysis in the single year 1986, which is remarkable.

Mizan also worked with Sergei Suslov, leading to nine joint papers in the nineties. In particular, in two papers [15,16] they studied in a general fashion the relation between the solution of the Pearson equation and (q-)beta integrals, giving a uniform treatment of many known and classical results as well as new results. In another paper [2] by Mizan and Sergei, jointly with Richard Askey, many bilinear generating functions and Poisson kernels for orthogonal polynomials from the q-analogue of the Askey scheme are given. This leads to many integral transforms

generalizing the Fourier transform, which corresponds to the Mehler formula, or the Poisson kernel for the Hermite polynomials, leading to the standard Fourier transform. For some reason, the paper [2] is not listed in the publication list of Mizan as given in [8]. Mizan's last mathematical paper is 121, which will appear soon and was written jointly with Mourad Ismail. It deals with instances when the parameters in the Askey scheme have critical values that make the polynomial of degree n factor as product of two polynomials. This approach was started by F. Calogero.

In addition to the book *Basic Hypergeometric Series*, George Gasper and Mizan have many joint papers, including several papers with important results on positivity for Poisson kernels for polynomials in the q-Askey scheme, in particular the q-Racah polynomials. For several classes of orthogonal polynomials from the (q-)Askey scheme they also calculated explicit expressions for linearization and connection coefficients in terms of very-well poised basic ${}_{10}\varphi_9$ and ${}_8F_7$ hypergeometric series. Moreover, in the collaboration with George Gasper a study of several orthogonal polynomials of two variables was undertaken. In Mizan's work with Mike Hoare these polynomials and their results also found a probabilistic interpretation.

Mizan was a very gentle and generous person. He was very kind to younger mathematicians, and he was always willing to help to crack a problem when he was called upon. Being a modest person, he was very hesitant to have his contribution acknowledged as e.g. a coauthor, which has been witnessed by the authors on a number of occasions. The same kindness and generosity was shown by Mizan to his family. He cared for his wife Parul for a long time, while she was suffering from kidney disease. He also spent a lot of time on the music career of their son Raja, including driving him to many music competitions and lessons.

As an author of novels and short stories in Bengali, Mizan was a well-known figure in the Bengali community in North America. Many of his stories have been collected and published in books, and some of them have been translated. The stories deal with a wide variety of subjects, such as emigration, old age, humanism, essays on science, social conditioning, etc. Mizan expressed a humanistic point of view to these issues, and he struck a chord with many of his readers. His columns are still very popular, and some of his remaining columns will be translated and published. At the moment, you can buy the original Bengali version of one of his books "Brihottor Barisaler Lokosonskrito" (Folklore of greater Barisal) at Amazon.com, see also [6]. Mizan has published twelve books, both from Dhaka and Kolkota. Another publication was in the works at the time of his passing. As with his mathematical writings, Mizan likewise touched millions of Bengali readers. "Shunno" ("Zero") is perhaps one of his most acclaimed publications. One of his most ardent fans had the following to say about him: "Knowing Mizan Uncle was like stumbling upon a pearl while taking a walk in the woods. He wrote about pure reason without losing the eternal respect for the human kind. He wrote many personal stories together with his scientific articles—a man of reason with a loving heart came through in every one of them. He was brutally honest yet respectful and loving to life itself". Additional material is in [8].

The following story shows how famous Mizan was amongst the Bengali community in North-America. During a visit of one of us to Mizan in Ottawa, we went out for dinner at a local Indian restaurant. Apparently, most Indian restaurants in Ottawa are run by Bengali, and as soon as the waiter found out that the writer Mizan Rahman was dining, they all flocked around him. Signatures, all kinds of extra dishes, no bill; usually a treatment that famous sportsmen or artists get, but only very rarely a scientist. The exact same story happened when the other author visited Mizan. It was an amazing experience to see this happening to a modest mathematician, even though Mizan's rise to fame was not for his mathematical work. Mizan's literary work is in Bengali, and so unreadable for most of us. However, some of his stories or columns have been translated into English. From these few stories we can see that Mizan had a truly humanistic view

on life and that he held strong opinions on the influence of religion on societies and politics, in particular with regard to his home country Bangladesh. Note that at the time of Mizan's birth in 1932, current Bangladesh was part of the British Empire. With the independence from British rule, the division of the subcontinent along religious lines led to India and Pakistan, where the Muslims were concentrated and which consisted of East Pakistan and West Pakistan. In 1971, East Pakistan became independent from Pakistan and continued as Bangladesh. So the dominant religion of Bangladesh is Islam. It seems clear that the foundations of Mizan's humanistic point of view on life and its major issues go back to his experiences in (what is now) Bangladesh.

Dennis Stanton sent us the following comments:

"Mizan was a special delight to meet anywhere. It was always disappointing to me if I found out he was not in attendance at conferences. He had a big heart and a positive view of mathematics, with free exchanges of ideas. Mizan would help on any problem I brought to him. He had special basic hypergeometric skills that are gone. I am personally grateful for Mizan's support and concern when things were not going so well for me. Mizan led an admirable life, and I knew only a part of it".

Mizan passed away quite unexpectedly on January 5, 2015. It is with a sad feeling that we realize that we will not be able to take part in a conversation with Mizan anymore, neither on mathematics nor life's important issues. He will be missed.

Scientific publications by Mizan Rahman

A list of 100 research publications by Mizan Rahman is presented in [8, p. 11–18]. The list given here contains the one in [8], and extends and updates this list.

1. PhD-thesis appears as part of Chapter 6 of "Kinetic Equations of Gases and Plasmas" by Ta-You Wu, (Addison–Wesley, 1966), specifically pp. 187–193.
2. "Collisional relaxation in a hard-sphere gas" (with M.K. Sundaresan), Phys. Lett. **25A** (1967), 705–706.
3. "Discrete relaxation modes for a hard-sphere gas" (with M.K. Sundaresan), Canad. J. Phys. **46** (1968), 2463–2469.
4. "Continuum eigenfunctions for a hard-sphere gas" (with M.K. Sundaresan), Canad. J. Phys. **46** (1968), 2287–2290.
5. "Continuum eigenfunctions in the neighborhood of singularities for a uniform hard-sphere gas", Canad. J. Phys. **48** (1970), 151–153.
6. "The maximum likelihood estimate of the noncentrality parameter of a noncentral F variate" (with J.N. Pandey), SIAM J. Math. Anal. **2** (1971), 269–276.
7. "On the integrability and application of a generalized Riccati equation", SIAM J. Appl. Math. **21** (1971), 88–94.
8. "A characterization of the exponential distribution" (with M. Ahsanullah), J. Appl. Prob. **9** (1972), 457–461.
9. "A singular inverse of a matrix by rank annihilation" (with M. Ahsanullah), Canad. Math. Bull. **15** (1973), 1–4.
10. "A note on the exposed values of powers of a matrix" (with M. Ahsanullah), Canad. J. Statist. **1** (1973), 123–125; erratum, ibid. **2** (1974), 284.
11. "On the spectral theory of Rayleigh's piston I. The discrete spectrum" (with M.R. Hoare), J. Phys. A **6** (1973), 1461–1478.
12. "Explicit form of the distribution of the Behrens–Fisher d-statistic" (with A.K.Md. Ehsanes Saleh), J. Roy. Stat. Soc. B. **36** (1974), 54–60; corrigendum, ibid. **36** (1974), 466.

13. "Bounds for expected values of order statistics" (with A.B.M. Lutful Kabir), Comm. Stat. **3** (1974), 557–566.
14. "On the spectral theory of Rayleigh's piston II. The exact singular solution for unit mass ratio" (with M.R. Hoare), J. Phys. A **7** (1974), 1070–1093.
15. "On the spectral theory of Rayleigh's piston III. Exact solution of the absorbing barrier problem ($\gamma = 1$)" (with M. R. Hoare), J. Phys. A **9** (1976), 77–85.
16. "Construction of a family of positive kernels from Jacobi polynomials", SIAM J. Anal. **7** (1976), 92–116.
17. "A five-parameter family of positive kernels from Jacobi polynomials", SIAM J. Math. Anal. **7** (1976), 386–413.
18. "Some positive kernels and bilinear sums for Hahn polynomials", SIAM J. Math. Anal. **7** (1976), 414–435.
19. "Exact transform solution of the one-dimensional special Rayleigh problem" (with J.A. Barker, M.R. Hoare and S. Raval), Canad. J. Phys. **55** (1977), 916–928.
20. "Stochastic processes and special functions: On the probabilistic origin of some positive kernels associated with classical orthogonal polynomials" (with R.D. Cooper and M.R. Hoare), J. Math. Anal. Appl. **61** (1977), 262–291.
21. "On a generalization of the Poisson kernel for Jacobi polynomials", SIAM J. Math. Anal. **8** (1977), 1014–1031.
22. "A generalization of Gasper's kernel for Hahn polynomials: application to Pollaczek polynomials", Canad. J. Math. **30** (1978), 133–146.
23. "A positive kernel for Hahn–Eberlein polynomials", SIAM J. Math. Anal. **9** (1978), 891–905.
24. "An elementary proof of Dunkl's addition theorem for Krawtchouk polynomials", SIAM J. Math. Anal. **10** (1979), 438–445.
25. "Distributive processes in discrete systems" (with M.R. Hoare), Physica A **97** (1979), 1–41.
26. "A product formula and a non-negative Poisson kernel for Racah–Wilson polynomials", Canad. J. Math. **30** (1980), 1501–1517.
27. "A stochastic matrix and bilinear sums for Racah–Wilson polynomials", SIAM J. Math. Anal. **12** (1981), 145–160.
28. "Families of biorthogonal rational functions in a discrete variable", SIAM J. Math. Anal. **12** (1981), 355–367.
29. "A non-negative representation of the linearization coefficients of the product of Jacobi polynomials", Canad. J. Math. **33** (1981), 915–928.
30. "On the q-analogues of some transformations of nearly-poised hypergeometric series" (with B. Nassrallah), Trans. Amer. Math. Soc. **268** (1981), 211–229.
31. "Discrete orthogonal systems with respect to Dirichlet distribution", Utilitas Math. **20** (1981), 261–272.
32. "The linearization of the product of continuous q-Jacobi polynomials", Canad. J. Math. **33** (1981), 961–987.
33. "Reproducing kernels and bilinear sums for q-Racah and q-Wilson polynomials", Trans. Amer. Math. Soc. **273** (1982), 483–508.
34. "The Rayleigh model: singular transport theory in one dimension" (with M.R. Hoare and S. Raval), Phil. Trans. Roy. Soc. London A **305** (1982), 383–440.
35. "Positivity of the Poisson kernel for continuous q-ultraspherical polynomials" (with G. Gasper), SIAM J. Math. Anal. **14** (1983), 409–420.

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37. "Cumulative Bernoulli trials and Krawtchouk processes" (with M.R. Hoare), Stochastic Processes Appl. **16** (1983), 113–139.
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39. "A simple evaluation of Askey and Wilson's q-beta integral", Proc. Amer. Math. Soc. **92** (1984), 413–417.
40. "Projection formulas, a reproducing kernel and a generating function for q-Wilson polynomials" (with B. Nassrallah), SIAM J. Math. Anal. **16** (1985), 186–197.
41. "An infinite series with products of Jacobi polynomials and Jacobi functions of the second kind" (with M.J. Shah), SIAM J. Math. Anal. **16** (1985), 859–875.
42. "Sums of products of ultraspherical functions" (with M.J. Shah), J. Math. Phys. **26** (1985), 627–632.
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48. "A product formula for the continuous q-Jacobi polynomials", J. Math. Anal. Appl. **118** (1986), 309–322.
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