DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 9 - Notes**

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1. Consider the linear systems
$$\frac{dY}{dt} = AY.$$

   (a)
```
>> A=[2 -2;1 4];
>> [v,d]=eig(A)
v =
   0.8165               0.8165
  -0.4082 - 0.4082i  -0.4082 + 0.4082i
d =
   3.0000 + 1.0000i        0
        0               3.0000 - 1.0000i
>> v(:,1)/real(v(2,1))
ans =
  -2.0000
   1.0000 + 1.0000i
```
   A complex solution will be
$$e^{3t}(\cos t + i\sin t)\begin{pmatrix} -2 \\ 1+i \end{pmatrix}.$$
   Find the real and imaginary parts and the general real solution will be
$$c_1e^{3t}\begin{pmatrix} -2\cos t \\ \cos t - \sin t \end{pmatrix} + c_2e^{3t}\begin{pmatrix} -2\sin t \\ \cos t + \sin t \end{pmatrix}.$$
   The original is spiral source anticlockwise.

   (b)
```
>> A=[0 10;-1 2]
>> [v,d]=eig(A)
v =
   0.9535               0.9535
   0.0953 + 0.2860i   0.0953 - 0.2860i
d =
   1.0000 + 3.0000i        0
        0               1.0000 - 3.0000i
>> v(:,1)/real(v(2,1))
ans =
  10.0000
   1.0000 + 3.0000i
```
   A complex solution will be
$$e^{t}(\cos 3t + i\sin 3t)\begin{pmatrix} 10 \\ 1+3i \end{pmatrix}.$$
   Find the real and imaginary parts and the general real solution will be
$$c_1e^{t}\begin{pmatrix} 10\cos 3t \\ \cos 3t - 3\sin 3t \end{pmatrix} + c_2e^{t}\begin{pmatrix} 10\sin 3t \\ 3\cos 3t + \sin 3t \end{pmatrix}.$$
   The original is spiral source clockwise.

2. Consider the linear systems

$$\frac{dY}{dt} = AY,$$

where $Y = (x, y, z)^T$. Find the eigenvalues and eigenvectors.

(a)
```
>> A=[2 2 0;1 3 0;0 0 1];
>> [v,d]=eig(A)
v =
   -0.8944   -0.7071         0
    0.4472   -0.7071         0
         0         0    1.0000
d =
     1     0     0
     0     4     0
     0     0     1
>> v(:,1)/v(2,1)
ans =
    -2
     1
     0
>> v(:,2)/v(1,2)
ans =
     1
     1
     0
```

If $Y(0) = (3, 1, 2)^T$, x, y, z will all increase to ∞, tending towards the plane $y = x$. If $Y(0) = (-1, -2, -1)^T$, x, y, z will all decrease to $-\infty$, tending towards the plane $y = x$.

(b)
```
>> A=[3 -2 0;2 -2 0;0 0 -3]
>> [v,d]=eig(A)
v =
    0.8944    0.4472         0
    0.4472    0.8944         0
         0         0    1.0000
d =
     2     0     0
     0    -1     0
     0     0    -3
>> v(:,1)/v(2,1)
ans =
     2
     1
     0
>> v(:,2)/v(1,2)
ans =
    1.0000
    2.0000
         0
```

If $Y(0) = (3, 1, 2)^T$, x, y will tend to ∞, tending towards the plane $y = 0.5x$, as z decreases and tends to 0. If $Y(0) = (-1, -2, -1)^T$, x, y, z will all increase to 0, in the plane $y = 2x$..