DEPARTMENT OF MATHEMATICS

**MATHS 260 Tutorial Week 9**

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1. Consider the linear systems

$$\frac{dY}{dt} = AY,$$

where

(a) $A = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$

(b) $A = \begin{pmatrix} 0 & 10 \\ -1 & 2 \end{pmatrix}$

For each system

- Use Matlab (command window) to set up $A$ (for example, `A=[2 -2;-1 3]`)
- Use Matlab to find the eigenvalues and eigenvectors (`[v,d]=eig(A)`). Note that Matlab find the eigenvectors so that their norm is 1. You may want to divide through by one of the components to make them easier to draw. Can try dividing the eigenvector by the smallest component (eg. `v(:,2)/.2425` or `v(:,2)/min(abs(v(:,2)))`)
- Write down the general solution of the linear system, and sketch the phase portrait
- Use `pplane` to draw the phase portrait and compare with your sketch. Classify the equilibrium at the origin.

2. Consider the linear systems

$$\frac{dY}{dt} = AY,$$

where $Y = (x, y, z)^T$. For each of the $A$ matrices specified below, note that the $z$ component decouples from the first two. Find the eigenvalues and eigenvectors, sketch the $xy$-phase plane and the $z$-phase line. Describe the path followed by a solution for which (i) $Y(0) = (3, 1, 2)^T$ and (ii) $Y(0) = (-1, -2, -1)^T$.

(a) $A = \begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 3 & -2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

You may use Matlab to help you.