DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 8 - Notes**

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1. Consider the linear systems

$$\frac{dY}{dt} = AY,$$

where

(a)
```
>> A=[2 -2;-1 3];
>> [v,d]=eig(A)
v =
   -0.8944    0.7071
   -0.4472   -0.7071
d =
     1     0
     0     4
```
General solution is

$$Y = c_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

In the long term both components tend to ∞.

(b)
```
>> A=[1 1;4 -2];
>> [v,d]=eig(A)
v =

    0.7071   -0.2425
    0.7071    0.9701
d =
     2     0
     0    -3
```
General solution is

$$Y = c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

In the long term both components tend to ∞ except those that start on the vector (1,-4) which tend to 0.

(c)
```
>> A=[-2 1;1 -2];
>> [v,d]=eig(A)
v =
  0.7071    0.7071
 -0.7071    0.7071
d =
    -3     0
     0    -1
```

General solution is

$$Y = c_1 e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

In the long term both components tend to 0.

2. From Matlab

```
>> A=[1 1 1;2 1 -1;-8 -5 -3];
>> [v,d]=eig(A)
v =
    0.0000    0.5571   -0.4216
    0.7071   -0.7428    0.5270
   -0.7071   -0.3714    0.7379
d =
    2.0000         0         0
         0   -1.0000         0
         0         0   -2.0000
```

General solution is

$$Y = c_1 e^{2t} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} -4 \\ 5 \\ 7 \end{pmatrix}.$$

3. From Matlab

```
>> B=[1 -1; 5 -3];
>> [v,d]=eig(B)
v =
   0.3651 + 0.1826i   0.3651 - 0.1826i
   0.9129             0.9129
d =
  -1.0000 + 1.0000i        0
        0            -1.0000 - 1.0000i
```

Notice that the eigenvalues (and eigenvectors) are complex. And that in the phase portrait there are not any straight line solutions. The system would have unique solutions for any initial values so solutions cannot cross.