DEPARTMENT OF MATHEMATICS

MATHS 260 Tutorial Week 8

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1. Consider the linear systems
$$\frac{dY}{dt} = AY,$$
where

   (a) $A = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$

   (c) $A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

   For each system

   - Use Matlab (command window) to set up A (for example, `A=[2 -2;-1 3]`)
   - Use Matlab to find the eigenvalues and eigenvectors (`[v,d]=eig(A)`). Note that Matlab find the eigenvectors so that their norm is 1. You may want to divide through by one of the components to make them easier to draw. Can try dividing the eigenvector by the smallest component (eg. `v(:,2)/.2425` or `v(:,2)/min(abs(v(:,2)))`)
   - Write down the general solution of the linear system, and sketch the phase portrait
   - Use `pplane` to draw the phase portrait and compare with your sketch. Describe the long term behaviour of the solutions.

2. Find the general solution of the system
$$\frac{dY}{dt} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{pmatrix} Y.$$
   You may use Matlab to find the eigenvalues and eigenvectors.

3. Consider the system
$$\frac{dY}{dt} = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} Y.$$
   - Use Matlab (command window) to set up B, the matrix above
   - Use Matlab to find the eigenvalues and eigenvectors of B
   - Use `pplane` to draw the phase portrait. What do you notice?