DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 7**

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1. The following equations model the growth of a population of mice and a population of ferrets. $M(t)$ gives the size of the mouse population (measured in thousands) as a function of time, and $F(t)$ measures the size of the ferret population (in hundreds). Ferrets eat mice.

$$\begin{aligned}\frac{dM}{dt} &= 2M\left(1-\frac{M}{2}\right) - 1.2MF \\ \frac{dF}{dt} &= -F + 0.9MF\end{aligned}$$

   (a) What feature is being modelled by each term in the equations?

   (b) Describe the behaviour of the mouse population if the ferret population is extinct. (For example, sketch the phase line for the prey population assuming that predators are extinct, and sketch the graphs of the prey population as a function of time for several solutions. Then interpret these graphs for the prey population.)

   (c) Describe the behaviour of the ferret population if the mouse population is extinct.

   (d) Use `pplane` to show the slope field for the system. Show some solutions including that which corresponds to initial populations of 1000 mice and 200 ferrets. Describe the long-term fate of the mouse and ferret populations for initial populations of 1000 mice and 200 ferrets.

   (e) How would you modify the model to include the effect of hunting of the ferrets at a rate of 10 ferrets per unit time? Show on `pplane`. Describe the long-term fate of the mouse and ferret populations for initial populations of 1000 mice and 200 ferrets.

   (f) Suppose that the ferrets find another source of food that is limited. How would you modify the model to include this fact?

2. Two species represented by x and y live in the same area. The populations are modelled by

$$\begin{aligned}\frac{dx}{dt} &= x(2-0.4x-0.3y) \\ \frac{dy}{dt} &= y(1-0.1y-0.3x)\end{aligned}$$

   where the populations are measured in thousands and t in years.

   (a) What factors are being modelled in this system?

   (b) Find the equilibrium solutions.

   (c) Use `pplane` to show the slope field. Check your equilibrium solutions from part (a).

   (d) Find the long term behaviour of the populations in the following cases:

      i. $x(0) = 1.5$, $y(0) = 3.5$

      ii. $x(0) = 1$, $y(0) = 1$

      iii. $x(0) = 2$, $y(0) = 7$

      iv. $x(0) = 2.5$, $y(0) = 4.5$

   What do you notice about the equilibrium point where both populations are non-zero?