DEPARTMENT OF MATHEMATICS

MATHS 260 Tutorial Week 6

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1. Find the equilibrium solutions of the system of equations

$$\begin{aligned} \frac{dx}{dt} &= x + y \\ \frac{dy}{dt} &= x - x^3 - y \end{aligned}$$

Show the slope field using `pplane`, and use it to find the equilibrium solutions. This is done under the `Solutions` menu - choose `Find an equilibrium point` - then click the cursor near a possible equilibrium point. Do these equilibrium points agree with your answers?

2. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= 2x - y + 3(x^2 - y^2) + 2xy \\ \frac{dy}{dt} &= x - 3y - 3(x^2 - y^2) + 3xy \end{aligned}$$

   (a) Use `pplane` to show the slope field. Note that this is the default equation system for `pplane`.

   (b) Show the equilibrium solutions.

   (c) Under `Options/Solver` choose `Runge-Kutta 4`.

      i. Show a solution satisfying $(x(0), y(0)) = (0, 2)$.

      ii. Show a solution satisfying $(x(0), y(0)) = (2, 0)$.

      iii. Under `Options/Settings` change the stepsize to 0.01, and again show a solution satisfying $(x(0), y(0)) = (0, 2)$.

      iv. With the stepsize kept at 0.01, again show a solution satisfying $(x(0), y(0)) = (2, 0)$.

   (d) Under `Options/Solver` choose `Dormand Prince` and show solutions satisfying:

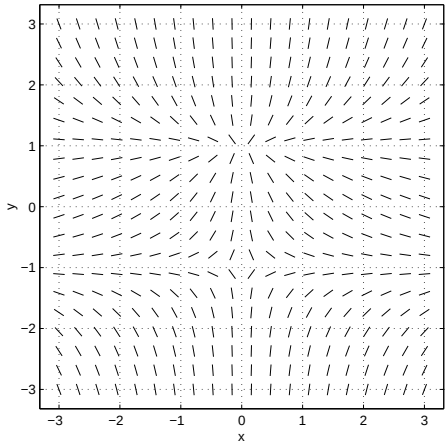
      i. $(x(0), y(0)) = (0, 2)$

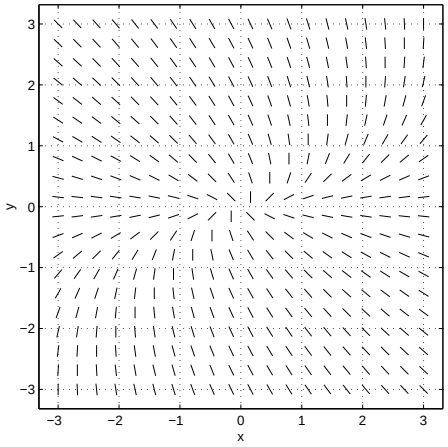
      ii. $(x(0), y(0)) = (2, 0)$

   Note that the Runge-Kutta method uses the same stepsize (in t) for each step, whereas Dormand Prince changes the stepsize during the calculation so that the error is controlled.

See over for some non-computer questions.

3. Match the slope fields shown below with one of the following systems of differential equations.





(a) $\dfrac{dx}{dt} = x, \quad \dfrac{dy}{dt} = y^2 - 1$ (b) $\dfrac{dx}{dt} = x - y, \quad \dfrac{dy}{dt} = 2y$

(c) $\dfrac{dx}{dt} = -x, \quad \dfrac{dy}{dt} = y^2 - 1$ (d) $\dfrac{dx}{dt} = x, \quad \dfrac{dy}{dt} = x + y$

4. Sketch the direction field corresponding to the following system of differential equations.

$$\frac{dx}{dt} = -2x, \quad \frac{dy}{dt} = -y.$$