DEPARTMENT OF MATHEMATICS

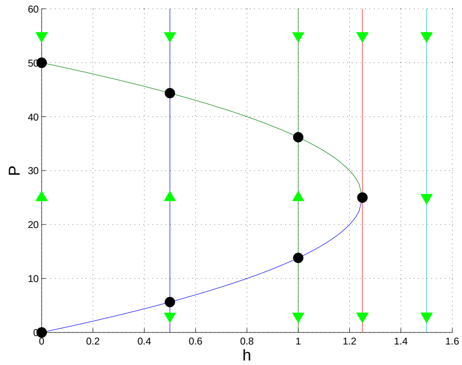
MATHS 260 Tutorial Week 5 - Notes

1. (a)

$$\frac{dP}{dt} = 0.1P(1 - \frac{P}{50}) - h.$$

(b) For $h = 1$, there are 2 equilibrium solutions (as we found last week). When $h = 2$, all the solutions are decreasing, so the population is not sustainable. Now look at some values of h between 1 and 2. For $h = 1.5$, it is not sustainable either. Try $h = 1.3$. Still not sustainable but getting better. Try $h = 1.2$. For this value, there appear to be equilibrium solutions at $P = 20$ and $P = 30$. Try $h = 1.25$. For this value, there appears to be an equilibrium solution at $P = 25$. Choose $h = 1.25$. The initial population must be at least 25,000.

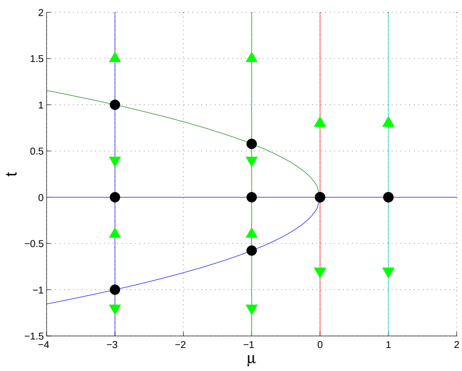
(c) The equilibrium solutions in terms of h are $P = 25 \pm \sqrt{1 - 0.8h}$. And $\sqrt{1 - 0.8h}$ exists only if $h < 1.25$.



2. (a) Phase line for $\mu = -3$ has a sink at 0 and sources at both -1 and 1. Phase lines for both $\mu = 0$ and $\mu = 3$ have only a source at 0.

(b) By trial and error using `dfield`, or by analysing the DE we find that the change is at $\mu = 0$.

(c) The bifurcation diagram is



3. (a) It is linear so use the integrating factor t^2. Solution is

$$y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{c}{t^2}.$$

(b) Tends towards the parabola, $y = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2}$.

(c) Solution to the IVP is

$$y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{1}{12t^2}.$$