DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 5**

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1. (Cf. Tutorial Week 4) Without harvesting a small population would initially grow at 10% per year, but the carrying capacity of the environment (i.e. the maximum possible number) is 50,000. Suppose h (in thousands) are harvested each time period.

   (a) Write the differential equation to model the population measured in thousands.

   (b) Use `dfield` to estimate which values of h will allow the population to be sustainable. Estimate the initial population required.

   (c) Sketch the bifurcation diagram.

2. Consider the one-parameter family of differential equations

$$\frac{dy}{dt} = y(\mu + 3y^2).$$

   (a) Use `dfield` to show the slope field for the following values of μ. From this draw the phase line (by hand) for each μ.

      i. $\mu = -3$

      ii. $\mu = 0$

      iii. $\mu = 3$

   (b) Can you find the value of μ where the change in qualitative behaviour occurs?

   (c) Sketch the bifurcation diagram.

3. Find a one-parameter family of solutions for the differential equation

$$t\frac{dy}{dt} + 2y = t^2 - t + 1.$$

   (a) What you you expect will happen as $t \to \infty$? Check your answer using `dfield`.

   (b) Find a solution to the IVP

$$t\frac{dy}{dt} + 2y = t^2 - t + 1, \quad y(1) = 0.5.$$