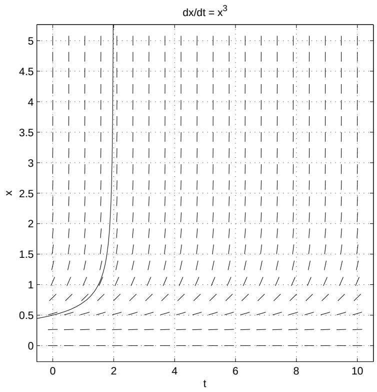
DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 4 - Notes**

1.
$$\frac{dy}{dt} = y^3, \quad y(0) = 0.5.$$

(a) $f(t, y) = y^3$ which is continuous at (0,0.5) so there is a solution. Also $frac\partial f\partial y = 3y^2$ which is also continuous at (0,0.5) so there is a unique solution.

(b) It seems that the solution exists for $t < 2$.



(c) The DE is separable and the general solution is

$$y = \pm\frac{1}{\sqrt{2(-t-c)}}.$$

When the initial condition is substituted in, we find that we must use the + sign and $c = -2$ so the solution is

$$y = \frac{1}{\sqrt{2(2-t)}},$$

and we can see that the solution exists for $t < 2$.

2. (a)
$$\frac{dP}{dt} = kP(1 - \frac{P}{N}).$$

(b)
$$\frac{dP}{dt} = kP(1 - \frac{P}{N}) - H.$$

(c) For a small value of P, we have approximately

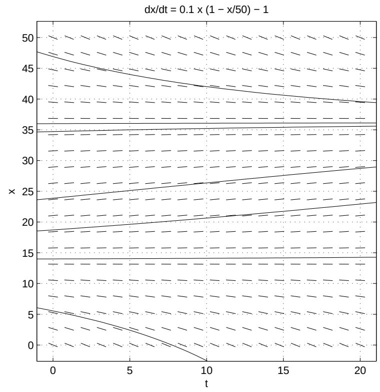
$$\frac{dP}{dt} = kP$$

and so we choose $k = 0.1$. Since the maximum population is 50,000, we set $N = 50$ (expressing the population in thousands). DE is

$$\frac{dP}{dt} = 0.1P(1 - \frac{P}{50}.$$

There are equilibrium solutions at 0 and 50. *Draw your own phase line.*

(d)
$$\frac{dP}{dt} = 0.1P(1 - \frac{P}{50} - 1.$$

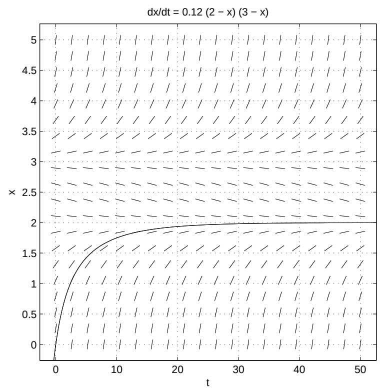


The equilibrium solutions are at approximately 14 and 36. *Draw your own phase line.*

3. Proportionally, molecules and moles are the same, so we can write the equation for X as if it were in molecules then call it moles.

(a)
$$\frac{dX}{dt} = k(\alpha - X)(\beta - X).$$

(b)
$$\frac{dX}{dt} = 0.12(2 - X)(3 - X).$$



After a long time there will be 2 moles of C, none of A and 1 of B.