DEPARTMENT OF MATHEMATICS

MATHS 260 Tutorial Week 4

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1. Consider the initial value problem

$$\frac{dy}{dt} = y^3, \quad y(0) = 0.5.$$

   (a) Show that this IVP has a unique solution.

   (b) Use `dfield` to show the solution. For what values of t do you think the solutions exists?

   (c) Solve the IVP and check your answer to (b).

2. This question is about the effect of harvesting on a population.

   (a) Write down a differential equation to model a population with bounded growth.

   (b) Modify your differential equation to take account of a constant number being harvested every time period.

   (c) Without harvesting a small population would initially grow at 10% per year, but the carrying capacity of the environment (i.e. the maximum possible number) is 50,000. Write the differential equation to model the population measured in thousands. Find the equilibrium solutions. What type of equilibrium is each? Draw a phase line.

   (d) If 1000 are harvested every year, write down a differential equation to model the population measured in thousands. Use `dfield` to show the slope field, and estimate the equilibrium populations. What type of equilibrium is each? Draw a phase line.

3. Two chemicals A and B combine to form chemical C. One molecule of A with one molecule of B yields one molecule of C (A + B $\longrightarrow$ C). The rate of production of C is proportional to product of the amount of A left and the amount of B left. Initially there is α of A and β of B and no C. Let X be the amount of C formed. All amounts are measured in moles.
   Note that:

   - *A mole is that quantity of a substance whose mass in grams is the same as its formula weight. For example, iron has an atomic weight of 55.845, so a mole of iron weighs 55.845 grams.*
   - *One mole of A with one mole of B yields one mole of C*

   (a) Write a differential equation to model X, the amount of C.

   (b) Assume the constant of proportionalilty is 0.12, there are initially 2 moles of A and 3 moles of B and no C. Use `dfield` to draw a slope field for the differential equation. After a long time, how much of each of the chemicals will be left?