DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 3**

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1. Consider the differential equation

$$\frac{dy}{dt} = 2y + t.$$

Use `dfield` to find a slope field for the differential equation.

(a) Try some initial values and see what happens to the solution as $t \to \infty$.

(b) Show the solution for initial condition $y(0) = -0.25$.

(c) If the initial condition is given in the form $y(t_0) = y_0$, can you predict the long term behaviour of the solution? How?

2. Consider the differential equation

$$\frac{dy}{dt} = 2y + t, \quad y(-2) = 1.$$

(a) Use `dfield` to find a slope field for the differential equation.

(b) Choose `Options/Solver/Euler`. When a window pops up, change the stepsize to 2. Choose `Options/Keyboard input`. Put the initial condition in the pop up window.

- Notice how the numerical solution is estimated by the slope on the slope field at the beginning of the step.
- Use the slope field to estimate the value of $y(0)$ that you would get if you used Euler's method with stepsize $h = 2$ to approximate the solution of the initial value problem at $t = 0$.

(c) By hand, use Euler's method with $h = 1.0$ to approximate the value of the solution to the initial value problem at $t = 0$.

(d) The solution to the initial value problem is

$$y(t) = -\frac{t}{2} - \frac{1}{4} + \frac{e^{2t+4}}{4}.$$

Use this fact to determine that the error in the approximation you obtained in (c). How could you adapt your method in (c) to obtain a more accurate approximation to the initial value problem?

(e) Use `numerical` to estimate $y(0)$ using Euler's method. For the "solution at final t", use `(exp(4)-1)/4`. What do you notice about the errors shown? What do you notice about the effective order shown?

(f) Repeat (d) for Improved Euler and 4th order Runge-Kutta.

Please turn over

3. Consider the initial value problem

$$\frac{dy}{dt} = t^2 - y^2, \quad y(0) = 2.$$

For each of the methods

- 4th order Runge-Kutta
- Improved Euler
- Euler

use `numerical` to estimate how many steps are required for the approximation of $y(0.4)$ to be accurate to 3 d.p. Check 4th order Runge-Kutta first to get an idea of the exact result. Use these results to find how many times, in total, the right side of the differential equation must be calculated for each method.