DEPARTMENT OF MATHEMATICS

MATHS 260 Tutorial Week 2

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1. Determine whether any of the following functions is a solution to the differential equation

$$t\frac{dy}{dt} - y = t^2.$$

(a)
$$y_1(t) = t^2$$

(b)
$$y_2(t) = 5t$$

(c)
$$y_3(t) = t^2 + 3t$$

2. (a) Find a one-parameter family of solutions (i.e. an expression containing one arbitrary constant) to the differential equation

$$\frac{dy}{dt} = -y^3(t+1).$$

(b) Are any solutions to the differential equation not contained in your family? Give a reason for your answer.

(c) Find a solution to the differential equation which satisfies the initial condition:

i. $y(0) = 1$
ii. $y(0) = 2$
iii. $y(0) = -1$

(d) Use `analyzer` to plot the solutions in (2c).

(e) Use `dfield` to plot the slope field for the differential equation. Using `Options/Keyboard Input`, input the initial conditions in (2c). Compare these solutions with your plot from `analyzer`.

3. The rate of decay of a radioactive substance is proportional to the amount of the substance remaining. For radium-226, assume the constant of proportionality is 0.0004332 when time is measured in years and the amount in grams. Also assume we start with 0.5 grams.

(a) Write an initial value problem to model the decay.

(b) Use `dfield` to draw a slope field for the differential equation. Input the initial condition to draw a solution for the initial value problem.

(c) Use your slope field to estimate how much will remain after 5000 years.

(d) Use your slope field to estimate when there will be 0.4 grams remaining.