

1. (a) See lecture notes for explanation of model.
(b) See lecture notes
(c)
 - i. Equilibrium is a centre. The mass oscillates for ever.
 - ii. Equilibrium is a spiral sink. The mass oscillates with decreasing amplitude and approaches rest. It is underdamped.
 - iii. Equilibrium is a sink. The mass approaches rest without oscillation. It is overdamped.
 - iv. Equilibrium is a sink. The mass approaches rest without oscillation. It is critically damped.
2. $y = c_1 e^{2t} + c_2 e^{-3t}$.
(a) $y = \frac{3}{5}e^{2t} + \frac{2}{5}e^{-3t}$. Tends to infinity.
(b) $y = e^{-3t}$. Tends to zero.
3. (a) When $p = 3, q = 2, w = 1$,
 - i. $y = c_1 e^{-2t} + c_2 e^{-t}$.
 - ii. Substitute into DE to check.
 - iii. $y = c_1 e^{-2t} + c_2 e^{-t} + 0.1 \cos t + 0.3 \sin t$. Long term behaviour is periodic for ever.
(b) Oscillates for ever with a mixture of frequencies.
(c) When $q = w^2$.