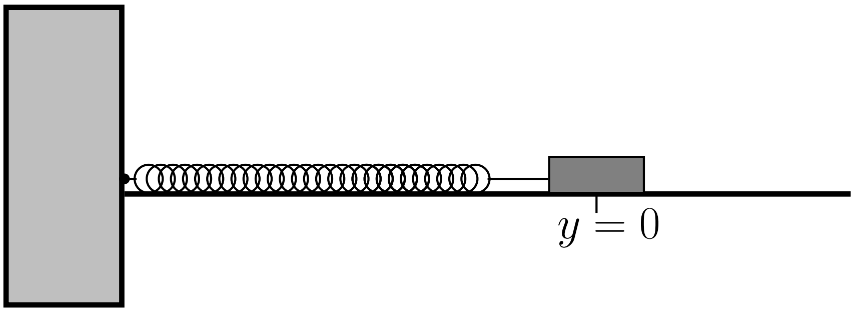
DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 12**

---

1. A mass is sitting on a table, attached to a spring which is attached to a wall.



The differential equation used to model the position of the mass can be written as

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0. \tag{1}$$

(a) Explain why this is a suitable model. Include a description of the parameters m, b and k.

(b) Write the differential equation as a system of two differential equations.

(c) Use `pplane` to investigate the behaviour of the solution for the parameters given below. Classify the equilibrium type for the system, look at the graphs for y and y' and describe the behaviour of the mass predicted by the model.

i. $b = 0, m = 10, k = 8.1$

ii. $b = 4, m = 10, k = 8.1$

iii. $b = 30, m = 10, k = 8.1$

iv. $b = 18, m = 10, k = 8.1$

2. Find the general solution of

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 0.$$

Investigate the long term behaviour for the initial conditions

(a) $y(0) = 1, y'(0) = 0$

(b) $y(0) = 1, y'(0) = -3$

3. Consider the forced harmonic oscillator modelled by

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \cos wt. \tag{2}$$

(a) When $p = 3, q = 2, w = 1$,

i. Find the general solution to the *homogeneous* equation.

ii. Show that $0.1\cos t + 0.3\sin t$ is a particular solution.

iii. Write down the general solution to the non-homogeneous equation and describe its long term behaviour. You may show the solution using `forced_osc`.

(b) Use `forced_osc` to show the solution when $p = 0, q = 4, w = 1$.

(c) If $p = 0$ and $q > 0$, under what conditions will the solution tend to ∞ when $t \to \infty$?