DEPARTMENT OF MATHEMATICS

**MATHS 260** **Tutorial Week 11 - notes**

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1. Equilibrium solutions at (0,0) and (1,0). Now find the Jacobian

$$J = \begin{pmatrix} 0 & 1 \\ 1-2x & -1 \end{pmatrix}.$$

```
>> syms xx yy
>> J=[0 1;1-2*xx -1]
J =
[      0,      1]
[ 1-2*xx,     -1]
>> [v,d]=eig(J)
v =
[ -(1/2+1/2*(5-8*xx)^(1/2))/(-1+2*xx), -(1/2-1/2*(5-8*xx)^(1/2))/(-1+2*xx)]
[                                   1,                                   1]
d =
[ -1/2+1/2*(5-8*xx)^(1/2),                       0]
[                       0, -1/2-1/2*(5-8*xx)^(1/2)]
>> subs(d,xx,0)
ans =
    0.6180         0
         0   -1.6180
>> subs(v,xx,0)
ans =

    1.6180   -0.6180
    1.0000    1.0000
>> subs(d,xx,1)
ans =
  -0.5000 + 0.8660i        0
        0            -0.5000 - 0.8660i
```

At (0,0), there is a saddle, and at (1,0) a spiral sink.
The $x$-nullcline is $y = 0$ and the $y$-nullcline is $y = x(1-x)$. Note that the points in the regions between the nullclines all have the same sign for $x'$ and $y'$.

2. The equilibrium solutions are at (0,0), (2,0), (1,1), (-2,4). Now the Jacobian which can be found using Symbolic toolbox in Matlab.

```
>> J=jacobian([x*(2-x-y),y*(y-x^2)])
J =
[ 2-2*x-y,      -x]
[  -2*y*x, 2*y-x^2]
J1=subs(J,{x,y},{0,0})
J1 =
     2     0
     0     0
>> [v,d]=eig(J1)  % had to do (0,0) and (2,0) separately - didn't work otherwise
v =
```

```
     0     1
     1     0
d =
     0     0
     0     2
>> J2=subs(J,{x,y},{2,0})
J2 =
    -2    -2
     0    -4
>> [v,d]=eig(J2)
v =
    1.0000    0.7071
         0    0.7071
d =
    -2     0
     0    -4
>> [v,d]=eig(J);
>> subs(d,{x,y},{1,1})
ans =
    1.7321         0
         0   -1.7321
>> subs(v,{x,y},{1,1})
ans =
   -0.3660    1.3660
    1.0000    1.0000
>> subs(d,{x,y},{-2,4})
ans =
    8.7446         0
         0   -2.7446
>> subs(v,{x,y},{-2,4})
ans =
    0.2965   -0.4215
    1.0000    1.0000
```

3. *First system*

   The equilibrium solutions are at (0,0), (0,-1), (0,1).

```
>> J=jacobian([x*(y-1);y*(-1+x^2+y^2)])
J =
[          y-1,             x]
[        2*y*x, -1+x^2+3*y^2]
>> [v,d]=eig(J);
>> subs(d,{x,y},{0,1})
ans =
     2     0
     0     0
>> subs(d,{x,y},{0,0})
ans =
    -1     0
     0    -1
>> subs(d,{x,y},{0,-1})
ans =
```

```
     2     0
     0    -2
```

Expect a sink at (0,0) and a saddle at (0,-1). At (0,1) we cannot tell from the Jacobian, however it does appear to be like a saddle from the phase portrait from `pplane`.

*Second system*
The equilibrium solutions are at (0,0) and (0,-1); in addition there is a line of equilibria at $y = 1$.

```
>> J=jacobian([x*(y-1);y*(-1+y^2)])
J =
[      y-1,         x]
[        0, -1+3*y^2]
>> [v,d]=eig(J);
>> subs(d,{x,y},{0,-1})
ans =
[  2,  0]
[  0, -2]
>> subs(d,{x,y},{0,0})
ans =
[ -1,  0]
[  0, -1]
>> subs(d,{y},{1})
ans =
     2     0
     0     0
```

Expect a sink at (0,0) and saddle at (0,-1). At (0,1) the Jacobian is the same as from the other system in this question but for this system there *is* a line of equilibria (sources) at $y = 1$. This is confirmed by `pplane`.