DEPARTMENT OF MATHEMATICS

MATHS 260 **Tutorial Week 11**

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1. For the nonlinear system

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = x - x^2 - y.$$

   (a) Find
      i. the equilibrium solutions
      ii. the Jacobian

   (b) For each equilibrium solution: find the Jacobian, its eigenvectors and eigenvaues, and do a sketch of the phase portrait near that equilibrium. Classify the equilibrium.

   (c) Find the equations for the nullclines.

   (d) Use `pplane` to show the phase portrait. Use `Solutions/Find an equilibrium point` to show the equilibria. Observe the behaviour near these points. Does it fit with your analysis of the Jacobian there?

   (e) Use `Solutions/Show nullclines` in `pplane` to add the nullclines to the phase portrait. Observe the direction arrows on the nullcllines. Observe the direction arrows in the regions between the nullclines. What do you notice?

   (f) Choose an initial point in each region and click to show the solution through that point. Can you predict what will happen before you click?

2. Repeat Question 1 for the nonlinear system

$$\frac{dx}{dt} = x(2 - x - y)$$
$$\frac{dy}{dt} = y(y - x^2).$$

3. Compare the following nonlinear systems. Find the equilibrium solutions, Jacobians and eigenvalues and use `pplane` to investigate.

$$\frac{dx}{dt} = x(y - 1)$$
$$\frac{dy}{dt} = y(-1 + y^2 + x^2).$$

- AND -

$$\frac{dx}{dt} = x(y - 1)$$
$$\frac{dy}{dt} = y(-1 + y^2).$$