

1. Consider the linear systems

$$\frac{dY}{dt} = AY,$$

where

```
(a) >> A=[-2 -1;1 -4];
>> [v,d]=eig(A)
v =
    0.7071    0.7071
    0.7071    0.7071
d =
   -3     0
     0    -3
>> A+3*eye(2)
ans =
     1    -1
     1    -1
```

The eigenvector is $(1, 1)^T$. For the generalised eigenvector w , $(A - \lambda I)w = (1, 1)^T$, which means that $w_1 - w_2 = v_1 = 1$. Therefore choose $w = (1, 0)^T$ for example. Then the general solution is

$$Y = c_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \left(t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$$

The origin is a sink. There is one straight line solution $y = x$. Solutions starting under this line tend to the origin from positive values of x and y . Those starting above this line tend to the origin from negative values of x and y .

```
(b) >> A=[2 4;3 6];
>> [v,d]=eig(A)
v =
   -0.8944   -0.5547
    0.4472   -0.8321
d =
     0     0
     0     8
>> v(:,1)/v(2,1)
ans =
   -2
     1
>> v(:,2)/v(1,2)
ans =
    1.0000
    1.5000
```

The general solution is

$$Y = c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2 e^{8t} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

There is a line of equilibrium points along $y = -0.5x$ which are all sources. All the solutions are straight line parallel to $y = 3/2x$ moving away from the equilibrium points.

2. Consider the system

$$\frac{dY}{dt} = \begin{pmatrix} a & a \\ 1 & 2a \end{pmatrix} Y.$$

```
>> syms a
>> B=[a a; 1 2*a];
>> [v,d]=eig(B);
```

- For $a = 2$,

```
>> subs(d,a,2)
ans =
    4.7321    0
    0    1.2679
>> subs(v,a,2)
ans =
    0.7321   -2.7321
    1.0000    1.0000
```

The origin will be a source.

- For $a = -2$,

```
>> subs(d,a,-2)
ans =
   -3.0000 + 1.0000i    0
    0   -3.0000 - 1.0000i
>> subs(v,a,-2)
ans =
    1.0000 + 1.0000i    1.0000 - 1.0000i
    1.0000    1.0000
```

The origin will be a spiral sink, anti-clockwise.

- For $a = -8$,

```
>> subs(d,a,-8)
ans =
   -9.1716    0
    0   -14.8284
>> subs(v,a,-8)
ans =
    6.8284    1.1716
    1.0000    1.0000
```

The origin is a sink.

- For $a = 1/2$,

```
>> subs(d,a,1/2)
ans =
    1.5000    0
    0    0
>> subs(v,a,1/2)
ans =
    0.5000   -1.0000
    1.0000    1.0000
```

There are a line of equilibria along $y = -x$, which are sources.