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Tut WK 2

(1) (a) $\frac{dy_1}{dt} = 2t$

$$\begin{aligned} t \frac{dy_1}{dt} - y_1 &= t(2t) - t^2 \\ &= 2t^2 - t^2 \\ &= t^2 \end{aligned}$$

$\therefore y_1$ is a soln

(b) $\frac{dy_2}{dt} = 5$

$$\begin{aligned} t \frac{dy_2}{dt} - y_2 &= t(5) - 5t \\ &= 0 \end{aligned}$$

$\therefore y_2$ is not a soln

(c) $y_3(t) = t^2 + 3t$

$$\frac{dy_3}{dt} = 2t + 3$$

$$\begin{aligned} t \frac{dy_3}{dt} - y_3 &= t(2t + 3) - t^2 + 3t \\ &= 2t^2 + 3t - t^2 - 3t \\ &= t^2 \end{aligned}$$

$\therefore y_3$ is a soln

$$2(a) \frac{dy}{dt} = -y^3(t+1)$$

$$\int \frac{dy}{y^3} = -\int (t+1) dt$$

$$-\frac{1}{2y^2} = -\frac{1}{2}t^2 - t + C$$

$$\frac{1}{y^2} = t^2 + 2t + 2C$$

$$y = \pm \frac{1}{\sqrt{t^2 + 2t + k}}$$

$$k = 2C$$

(b) $y=0$ is also a soln since $\frac{dy}{dt} = 0$
 $+ -y^3(t+1) = 0.$

However there is no value of k that makes the family in (a) into $y=0$.

(c) (i) $y(0)=1$

$$1 = + \frac{1}{\sqrt{0+0+k}}$$

$$\therefore k=1$$

$$\pm y = + \frac{1}{\sqrt{t^2 + 2t + 1}}$$

(ii) $y(0)=2$

$$2 = + \frac{1}{\sqrt{0+0+k}}$$

$$\sqrt{k} = \frac{1}{2}$$

$$k = \frac{1}{4}$$

$$y = + \frac{1}{\sqrt{t^2 + 2t + 1/4}}$$

(c)(iii)

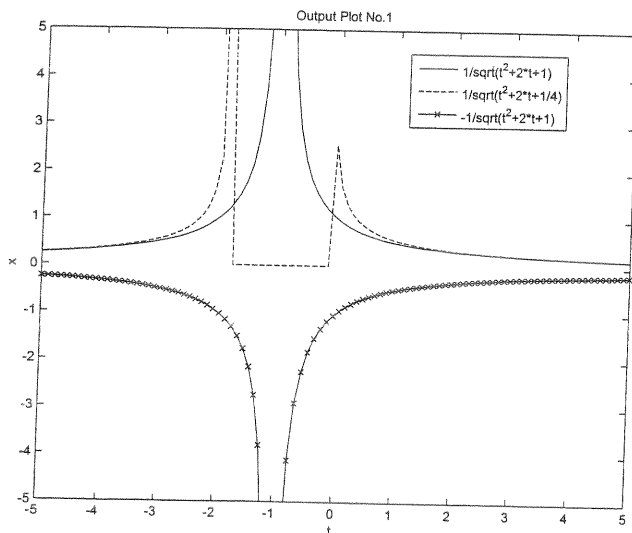
$$y(0) = -1$$

$$-1 = - \frac{1}{\sqrt{0+0+k}}$$

$$k = 1$$

$$y = - \frac{1}{\sqrt{t^2+2t+1}}$$

(d) Analyser



Notes:

(i) $y(0) = 1$

This soln will only be defined for $t > -1$.

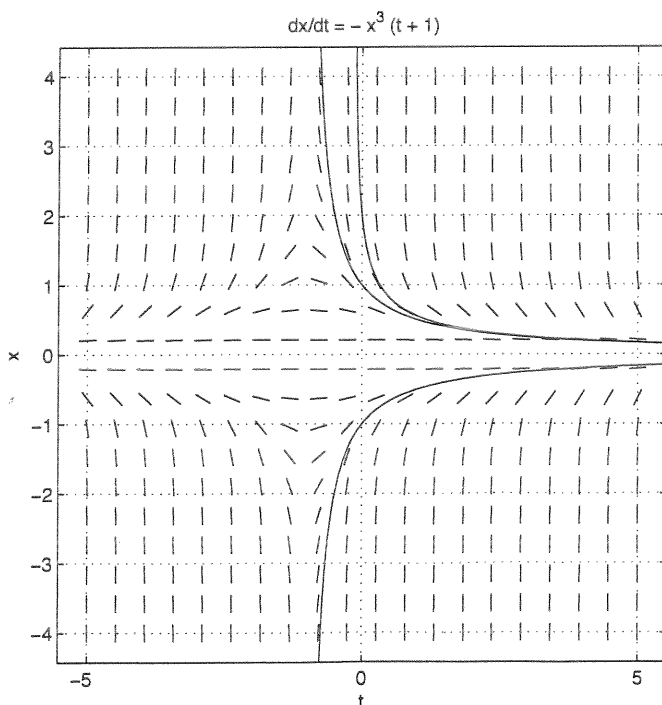
(ii) $y(0) = 2$

Only defined for $t > \frac{\sqrt{3}}{2} - 1$

(iii) $y(0) = -1$

Only defined for $t > -1$

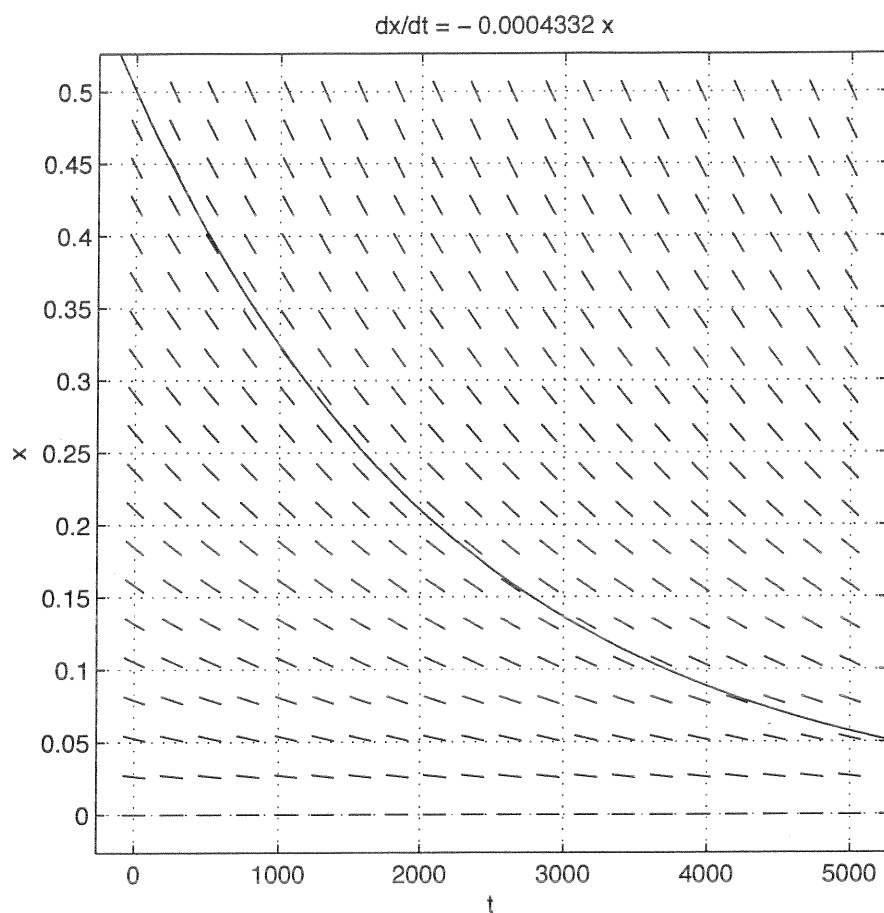
(e) dfield.



3. (a) Let $R(t)$ be the amount of the substance at time t .

Then $\frac{dR}{dt} = -0.0004332R$

(b)



(c) Approx 0.06g.

(d) Approx 500 years