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2004F Q7

$$\frac{dx}{dt} = -x(y+1)$$
$$\frac{dy}{dt} = -x-y+a$$

(a) $\frac{dx}{dt} = 0 \iff x=0$ or $y=-1$

$\frac{dy}{dt} = 0 \iff y = -x+a$

$\equiv$m solns are $(0, a)$ and $(1+a, -1)$

$$J(x,y) = \begin{pmatrix} -y-1 & -x \\ -1 & -1 \end{pmatrix}$$

$$J(0,a) = \begin{pmatrix} -a-1 & 0 \\ -1 & -1 \end{pmatrix}$$

Eigenvalues are -1 and $-a-1$.

If $a < -1$, saddle.

$a > -1$ sink

$a = -1$ no information (e/v $= 0$)

(b)
$$\frac{dx}{dt} = -x(y+1)$$
$$\frac{dy}{dt} = -x-y+1$$

(i) other $\equiv$m soln is $(1+a, -1) = (2, -1)$

$$J(2,-1) = \begin{pmatrix} 0 & -4 \\ -1 & -1 \end{pmatrix}$$

$$\det(J - \lambda I) = \det\begin{pmatrix} -\lambda & -4 \\ -1 & -1-\lambda \end{pmatrix} = \lambda(\lambda+1) - 4$$
$$= \lambda^2 + \lambda - 4$$

$$\lambda = \frac{-1 \pm \sqrt{1+4}}{2}$$

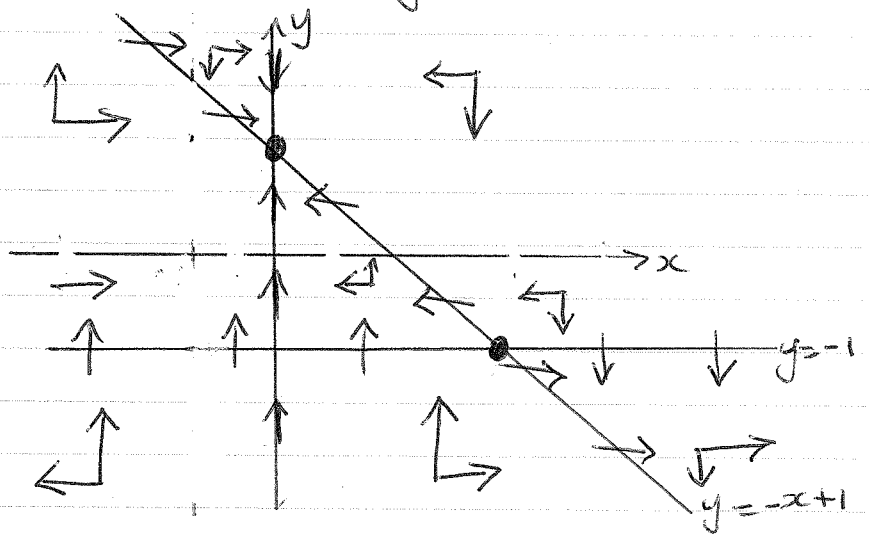
$$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

Opp signs $\Rightarrow$ saddle.

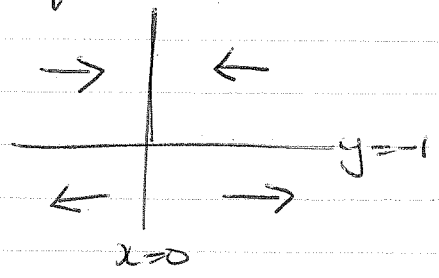
At (0,1) we have sink (as in (a))

(ii) x nullclines $x=0$, $y=-1$

y ~~nucl~~ nullcline $y=-x+1$



Look at 4 quadrants formed by $x=0$, $y=-1$



Also above $y=-x+1$ will have $y>-x+1$

ie $\frac{dy}{dt} < 0$ ↓

below $y=-x+1$, $\frac{dy}{dt} > 0$, ↑

Candidate's Name: _____ ID No: _____

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Answer sheet for Question 7(b)