**Department of Mathematics**
Maths 260 *Differential Equations*
**An introduction to software used in the course**

This handout provides information about using the software package *MATLAB* to investigate the behaviour of solutions to differential equations. The package allows you to enter your own equations; it will plot slope fields and find numerical approximations to solutions of your equations.

There are various functions in the *MATLAB* package. We will use four regularly: *analyzer*, *dfield*, *numerical* and *pplane*. To use one of these functions, first login to a terminal in the Mathematics and Statistics Computer Laboratory, then open *MATLAB*. (If you are unsure how to login or to open *MATLAB*, ask a demonstrator in the laboratory for help.)

Then you need to copy all the files in the 260 folder (look under course related folder on the desktop) into the H: drive (you will only need to do this once). Then you should set the directory for *MATLAB* to the H: drive (you will have to do this every time you start *MATLAB*). Then, for example, to start *analyzer*, type "`analyzer`" at the command prompt.

An easy way to learn how to use these functions is to work through the following exercises.

## 1 analyzer

Use *analyzer* to draw graphs (all on one set of axes) for the equations

$$x = c_1 \sin t + c_2 t$$

for $t = -8$ to $t = 8$ with the following values of $c_1$ and $c_2$

(a) $c_1 = 1$, $c_2 = 0$

(b) $c_1 = 0$, $c_2 = 1$

(c) $c_1 = 1$, $c_2 = 1$

(d) $c_1 = 0.8$, $c_2 = 0.2$

Print out the graphs.

*Procedure*

1. When the dialogue box comes up, put in the equation:

$$x = \sin(t)$$

Note: For example, to input the equation $x = \exp(t^2) + \frac{3}{2t}$, in the box after '`x(t)=`' enter

```
exp(t^2)+3/(2*t)
```

Remember to put '*' between 2 and t to represent the multiplication!

2. In the "Axis limits" box, change the ranges of values for $t$ and $x$ to those desired. An appropriate range for the $t$ values is given above but you will need to choose your own $x$ values.
3. Click the *Proceed* button.
4. To plot the next graph, go back to the *ANALYZER PLOT DETAILS* window and enter the next equation in place of the old.
5. Again click the *Proceed* button. Look at the *ANALYZER PLOT* window.
6. Repeat (4) and (5) for the remaining graphs.
7. If you wish to give the graph a title, enter the following into the *MATLAB* command window: title('graph of $x(t) = ...$')
8. To print the graph, first log onto *NetLogin.* Choose *Print...* in *Menu* from the *ANALYZER PLOT* window.
9. To quit *analyzer*, click the *Quit* button in the *ANALYZER PLOT DETAILS* window.

*Other functions you may try* :

On ANALYZER PLOT DETAILS window

i. Uncheck the "Overlay on current plot" box to plot the forwarding graphs on a new plot window(s).

ii. Type different numbers in the "No. of points to be plotted" box to change the accuracy of the plot. (greater the number of points, better the graph looks but takes longer time to plot)

On ANALYZER PLOT window

iii. From the "Menu", you can *copy* the current plot window as an image so it can be pasted in any image editing programs (eg. Paint Brush, Microsoft Word) or you can *quit* ANALYZER. (To print, see above)

iv. You can put the grid on the current plot window by clicking "Grid on/off" menu item.

v. You can zoom in by first choose the *Zoom in* from "Zoom" menu then left-clicking on the plot area, or click and drag to set an area to be zoomed into. You can zoom out by right-clicking on the **zoomed in** plot area, or by selecting *Zoom to original size* from "Zoom" menu.

## 2 dfield

Use *dfield* to draw a direction field for the differential equation

$$\frac{dx}{dt} = 0.5xt$$

for $t = -5 : 5$ and $x = -10 : 10$.

Plot a solution for the differential equation which satisfies the initial condition

$$x(2) = 5.$$

Print the graph.

*Procedure*

1. In *MATLAB*, type *dfield* and in the *differential equation* box, after '$x' =$' type

   ```
   0.5*x*t
   ```

2. Set the ranges of $t$ and $x$ in the *display window* as above. Click *Proceed*.
3. To draw in the particular solution, click on the initial condition point (2,5) in the $t - x$ plane. The solution through that point will be drawn. Watch the text in the lower left of the *PLOT* window to see if you have got the right starting point. If you are not satisfied with your solution, choose *Erase all solutions* from the *Edit* menu and try again.
4. To see which numerical method and step size have been used to plot the solution, choose *Solver Settings* from the *Options* menu. More information will be given about how these work later in the course.
5. Plot some other solutions to the differential equation by clicking on other initial condition points of your choice in the direction field.
6. Print the graph as in Question 1(h).
7. Use *Analyzer* to draw graphs (all on one set of axes) for the equations

   $$x = k\exp(t^2/4).$$

   for $t = -5 : 5$ and $x = -10 : 10$ with various values of $k$, for example, $k = \pm 1, \pm 0.5, 0.01$.

   Compare the graphs obtained from *dfield* with those from *analyzer*.

## 3 numerical

The *numerical* tool in the software package *MATLAB* can be used to calculate an approximate solution to an initial value problem using one of three numerical methods: Euler's method, Improved Euler's Method, and the 4th order Runge-Kutta method.

To use the tool, you must enter your differential equation, the initial values of the independent and dependent variables, and the final value of the independent variable. You can also enter the value of the solution at the final value of the independent variable if you know what it is. *MATLAB* will then calculate the approximate value of the solution at the final value of the independent variable for various choices of the stepsize. If you have provided a final value for the solution, the program will also calculate the error in each approximate solution and the effective order of the numerical method.

Work through the following example:

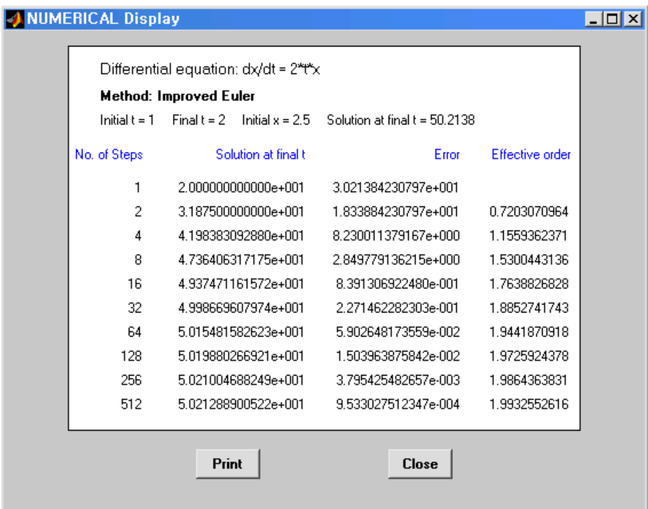
Use *numerical* to calculate an approximate solution to the initial value problem

$$\frac{dy}{dt} = 2ty, \quad y(1) = 2.5$$

at $t = 2$. Use the Improved Euler method. What is the effective order of the method at stepsize $h = 0.125$?

*Procedure*:

1. Open *numerical* and when the dialogue box comes up, type in the differential equation (note: you will have to use $x$ as the dependent variable).
2. Enter the initial and final values of $t$ (i.e., 1 and 2 respectively) and the initial value of $x$ (i.e. 2.5).
3. The formula for the solution to the IVP is $x(t) = 2.5 \exp(t^2 - 1)$. (You can obtain this solution by the method of separation of variables). Thus, the exact value of $x$ at $t = 2$ is $x(2) = 2.5 \exp((2)^2 - 1) = 2.5 \exp(3)$. Type $2.5 * \exp(3)$ in the box labelled 'solution at final $t$'.
4. Choose the numerical method Improved Euler and number of steps to be 8.
5. Click on *Proceed*, you should obtain the following output:


NUMERICAL Display

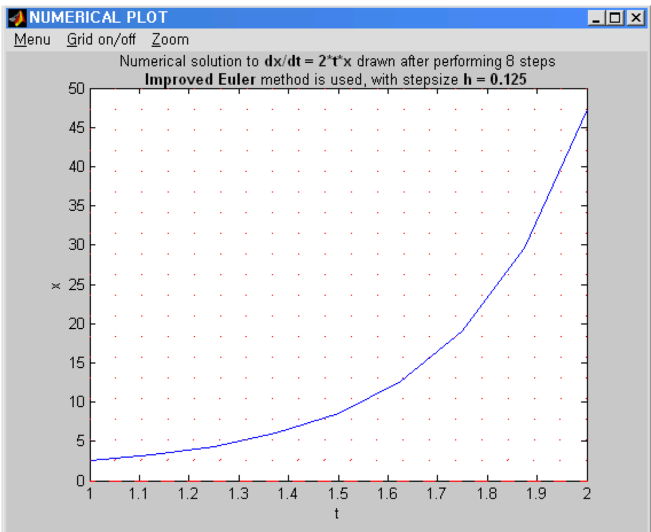
Differential equation: dx/dt = 2*t*x

**Method: Improved Euler**

Initial t = 1 Final t = 2 Initial x = 2.5 Solution at final t = 50.2138

| No. of Steps | Solution at final t | Error | Effective order |
|---|---|---|---|
| 1 | 2.000000000000e+001 | 3.021384230797e+001 | |
| 2 | 3.187500000000e+001 | 1.833884230797e+001 | 0.7203070964 |
| 4 | 4.198383092880e+001 | 8.230011379167e+000 | 1.1559362371 |
| 8 | 4.736406317175e+001 | 2.849779136215e+000 | 1.5300443136 |
| 16 | 4.937471161572e+001 | 8.391306922480e-001 | 1.7638826828 |
| 32 | 4.998669607974e+001 | 2.271462282303e-001 | 1.8852741743 |
| 64 | 5.015481582623e+001 | 5.902648173559e-002 | 1.9441870918 |
| 128 | 5.019880266921e+001 | 1.503963875842e-002 | 1.9725924378 |
| 256 | 5.021004688249e+001 | 3.795425482657e-003 | 1.9864363831 |
| 512 | 5.021288900522e+001 | 9.533027512347e-004 | 1.9932552616 |

Print Close



6. A stepsize of 0.125 means that $2^3 = 8$ steps are necessary to get from $t = 1$ to $t = 2$. From the output, we see that the approximate solution at $t = 2$ using 8 steps is 47.36 with an error of approximately 2.8. The effective order at this stepsize is given as approximately 1.5, i.e., halving the stepsize reduces the error by about $2^{1.5}$

7. You can also enable/disable plot fields on the NUMERICAL PLOT by checking or unchecking the "Draw field lines" box. Here you can also specify number of field segments (default is 20).

   Note: on NUMERICAL PLOT :

   (a) From the "Menu", you can *copy* the current plot window as an image so it can be pasted in any image editing programs (eg. Paint Brush, Microsoft Word) or *print* the graph.

   (b) You can put the grid on the current plot window by clicking "Grid on/off" menu item.

   (c) You can zoom in by first choose the *Zoom in* from "Zoom" menu then left-clicking on the plot area, or click and drag to set an area to be zoomed into. You can zoom out by right-clicking on the **zoomed in** plot area, or by selecting *Zoom to original size* from "Zoom" menu.

# 4 pplane

The function *pplane* from the software package *MATLAB* can be used to investigate the behaviour of solutions to systems of two, first order differential equations. The package allows you to enter your own equations; it will plot the slope field (which is just the direction field without arrows marked on the vectors) and find numerical approximations to solutions of your equations.

The following exercise is designed to help you get started using *pplane*.

Use *pplane* to draw the slope field for the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x + y),\\ \frac{dy}{dt} &= y(2 - x - 3y).\end{aligned}$$

Plot the phase portrait for the system of equations including the solution that passes through the point $(x, y) = (1, 2)$.

*Procedure*

1. Open *pplane* and enter the equations in the dialogue box.

2. Choose some appropriate values for the ranges of $x$ and $y$ by altering the entries in the appropriate boxes near the bottom of the window. For this example, suitable ranges are $x_{min} = -3, x_{max} = 3$, $y_{min} = -3, y_{max} = 3$.

3. Click the *Proceed* button.

4. Draw some solutions. To draw the solution that passes through the point $(x, y) = (1, 2)$ move the mouse until the arrowhead is at the point $(1, 2)$, then click the left mouse button. Use the dotted grid to help you locate the point you wish to click. The solution through that point will be drawn.

   The solution curve is usually plotted in two parts: first by starting at the initial point and letting time increase, then by starting at the initial point and letting time decrease. You should watch carefully to see which direction along the solution curve corresponds to increasing time, because when you have printed out your completed phase portrait you will need to draw an arrow on each solution curve to indicate the direction of increasing time.

5. Because you are not storing the trajectory information it is not possible to erase just one solution if you are not satisfied with it. However, you can erase all the solutions you have drawn so far by selecting *Erase all solutions* from the *Edit* menu. This will delete all the trajectories.

6. To see which numerical method and step size have been used to plot the solution and/or to alter the settings, choose *Settings* and *Solver* respectively from the *Options* menu. For this example, you will need to use a stepsize of less than 0.05 with the Runge-Kutta method to get a nice phase portrait. Numerical methods for systems of equations will be discussed in lectures.

7. Plot some other solutions to the differential equation by clicking on other initial points of your choice in the slope field.

8. You should be able to see approximately where the equilibrium points for the system are by looking for places where solutions move very slowly or places where the slope marks in the slope field change direction dramatically. For example, there seems to be an equilibrium point near $(x, y) = (1.25, 0.25)$. To determine the position of an equilibrium, first choose *Find an equilibrium point* from the *Solutions* menu then hold down the mouse button while you move the pointer to the approximate position of the equilibrium. Release the mouse button; a red circle will appear at the equilibrium point and the values of $x$ and $y$ at the equilibrium will be reported in a pop up window.

9. When you have a nice picture showing all the equilibrium points and some representative solution curves, print out your graph. Draw arrows on each solution curve showing the direction moved as time increases (see (4) above).