Maths 260 : An overview of the course

A differential equation expresses a relationship between a function and its derivatives (or some functions and their derivatives, in the case of a system of equations). For example, the DE

$$\frac{dx}{dt} = 3x$$

says the unknown function $x(t)$ is equal to three times its derivative.

The aim of this course is to learn ways to determine the properties of the functions that satisfy a given DE (i.e., properties of *solutions* to a DE). Information we seek about solutions may be:

1. qualitative information about solutions;
2. an explicit formula for a solution;
3. a numerical approximation to an explicit formula.

Before seeking a solution or information about a solution, we would like to be sure that a solution to the DE exists. The *Existence and Uniqueness Theorem* gives conditions under which solutions to a DE (or system of DEs) exist. These conditions turn out to be fairly general - solutions exist for a large class of DEs.

Finding Explicit Formulae for Solutions

Although the Existence and Uniqueness Theorem guarantees that solutions exist for many DEs, it is not usually possible to write down a formula for a solution to a DE. Special cases in which it is sometimes possible to find formulae for solutions include:

1. *Separable* or *linear* DEs with one dependent variable;
2. Linear, constant coefficient, autonomous systems of first order DEs;
3. Linear, constant coefficient, higher order DEs, either homogeneous or non-homogeneous. *Laplace transforms* can be used when the nonhomogeneous term is discontinuous.

Qualitative Information

A major technique for getting qualitative information about solutions to autonomous systems is to sketch the phase line or phase plane. This can involve:

1. finding equilibrium solutions;

2. classifying equilibrium solutions using linearisation or some other method;
3. using the slope field and/or nullclines.

In DEs that depend on a parameter, qualitative behaviour of solutions may change as the parameter varies. A *bifurcation diagram* can be useful for summarising the qualitative changes in the behaviour of solutions that occur as a parameter is varied.

Numerical Methods

Numerical approximations to explicit solutions at particular times can be found using a numerical integration method. We looked briefly at fixed stepsize methods:

- Euler's method
- Improved Euler's method
- 4th order Runge-Kutta method

Numerical methods can also be used to draw approximate phase portraits. We used the software package *Matlab* for this purpose.

Preparing for the Final Exam

To do well on the exam, you need to:

- understand the lecture material. Make sure you have a complete set of lecture notes and handouts. The textbook has good explanations of most topics and should be read in conjunction with your lecture notes.
- practise doing examples. Good sources of examples are old exams and tests, assignment and tutorial questions, and exercises in the text book.

Relevant material in the textbook is contained in: §§1.1-1.8, §§2.1-2.4, §§3.1-3.8, §§4.1-4.3, §§5.1-5.2, §§6.1-6.3, §§7.1-7.3.

Where to from here?

If you have enjoyed Maths 260 and want to do more courses in Applied Maths, consider:

Maths 361 and **Maths 362** : Advanced Methods in Applied Maths. These are core courses at Stage III. Prerequisite : B- or better in Maths 260 and Maths 253

Also **Maths 363** : Computational Mathematics and Modelling. Prerequisite: Either B+ or better in Maths 260 OR Maths 260 and Maths 270.

Maths 162 : Introduction to Applied and Computational Maths.

Maths 270 : Numerical Computation. Prerequisite : Maths 250 and Maths 162 (or course with equivalent programming content).

For help in selecting courses, see the Mathematics Department Undergraduate Handbook or speak to your lecturers.