Maths 260 Lecture 3

Topic for today
More on separable equations
Using the software provided with the textbook

Reading for this lecture
BDH Section 1.2 (again)

Suggested Exercises
BDH Section 1.2: 35, 40

Reading for next lecture
BDH Section 1.3

Today's handout
Lecture 3 notes

Example: Model of Student Loan

A student has a student loan of $20,000 when she completes her degree. For the next two years the student makes no repayments and the loan accumulates interest at 8% per year. Thereafter, the student pays off $3,600 per year and the interest remains at 8%. When will she finish paying off the loan?

Assumptions
interest rate constant
continuously compounding

Variables
L=size of loan (dependent variable) in $$
t=time (independent variable) in years

Model

Method of solution
We can regard this model as 2 DEs:

$$\frac{dL}{dt} = 0.08L, \qquad 0 \le t < 2, \tag{1}$$

$$\frac{dL}{dt} = 0.08L - 3600, \qquad 2 \le t. \tag{2}$$

Note that both equations are separable.

Case (1): $0 \le t < 2$

Case (2): $2 \leq t$

Using the software provided with the textbook

The second edition of the textbook ('Differential Equations' by Blanchard, Devaney and Hall) contains a CD, called 'DETools', of software useful for investigating differential equations. The programs on the CD are described in the Preface to the textbook. These programs are different from the Matlab routines you will mainly use in the computer laboratory to do assignment questions. If you have access to the CD, you can use it to investigate the behaviour of solutions to differential equations you see in lectures, in the textbook, and in assignments.

Revision Examples

1. Find the solution of the initial value problem

$$\frac{dy}{dt} = 2ty^2 + y^2, \qquad y(1) = \frac{1}{2}.$$

2. A population of kiwi and a population of stoats live in the same area. The stoats prey on the kiwi. A system of equations to model the population sizes is proposed as:

$$\frac{dP_1}{dt} = aP_1(1 - P_1 - bP_2)$$
$$\frac{dP_2}{dt} = cP_2 + dP_1P_2 - e$$

where a, b, c, d, e are positive constants.
Which populations do P_1 and P_2 represent? What factors have been included in this model?