Maths 260 Lecture 2

Topics for today
Getting started in the lab
Solutions to differential equations
Separable differential equations

Reading for this lecture
BDH Section 1.2

Suggested Exercises
BDH Section 1.2: 1, 3, 7, 15, 25

Today's handouts
Computer Laboratories for Mathematics and Statistics
An introduction to software used in the course
Lecture 2 notes

Getting started in the lab

There will be a tutorial to help you get started in the lab on **Monday, 6th March**.

This week you should

1. Make sure you know your NetAccount username and password
2. Find the lab
3. Book a computer
4. Login
5. Open 'Matlab'
6. Learn how to print

The Mathematics and Statistics Computer Laboratory is open Mon-Thu, 9am - 8pm and Friday, 9am - 5pm.

Section 1.2 Analytic Technique: Separation of Variables

Standard form for a first order DE is

$$\frac{dy}{dt} = f(t, y)$$

A solution of the DE is a function of the independent variable that, when substituted for the dependent variable in the DE, satisfies the DE for all values of the independent variable, i.e., $\phi(t)$ is a solution if $\frac{d\phi}{dt} = f(t, \phi)$ for all t.

Example

$$\frac{dy}{dt} = \frac{y^2 - 1}{t^2 + 2t}$$

Which of the following functions is a solution?

1. $y_1(t) = t + 1$

2. $y_2(t) = 1 + 2t$

3. $y_3(t) = 1$

Initial Value Problems

An **initial condition** tells us the value of a solution to a DE at a particular value of the independent variable. A DE with an initial condition is called an **Initial Value Problem (IVP)**.

e.g. $$\frac{dy}{dt} = te^{t^2} + 3, \qquad y(0) = -1$$

The function $y(t) = \frac{1}{2}e^{t^2} + 3t + c$ is a solution to the differential equation for all values of c, but only the choice $c = -3/2$ satisfies $y(0) = -1$.
The function $y(t) = \frac{1}{2}e^{t^2} + 3t + c$ is the **general solution** to the DE because we can use it to solve any IVP for this DE by correctly choosing c.

Separable Equations

It is usually not possible to find analytic solutions to a DE, but there are a few special cases when we can calculate explicit solutions. Separable equations are one such case.

A DE is called **separable** if

$$f(t, y) = g(t)h(y)$$

for some functions g and h.

Examples:

Special cases:

$$\frac{dy}{dt} = g(t)$$

$$\frac{dy}{dt} = h(y)$$

Solving Separable Equations
A separable DE can be written

$$\frac{dy}{dt} = g(t)h(y)$$

for some functions f and g. If $h(y) \neq 0$, divide by $h(y)$:

$$\frac{1}{h(y)}\frac{dy}{dt} = g(t)$$

Since y is a function of t, we get

$$\frac{1}{h(y(t))}\frac{dy}{dt} = g(t)$$

Integrate wrt t:

$$\int \frac{1}{h(y(t))}\frac{dy}{dt}dt = \int g(t)dt$$

By the chain rule, $dy = \frac{dy}{dt}dt$, so

$$\int \frac{1}{h(y)}dy = \int g(t).dt$$

If we can do these integrals we can get an expression for $y(t)$, the solution to the DE.

Examples

1.
$$\frac{dy}{dt} = t^3 y$$

Note that $y(t) = 0$ is also a solution to this DE but it is not found by this method ("missing solution").

2.
$$\frac{dy}{dt} = \frac{t}{1+y^2}$$

3.
$$\frac{dy}{dt} = e^{t^2}$$

Important ideas/words from today

solution to a DE
initial value problem
general solution
separable equation
autonomous equation
missing solution