Maths 260 Lecture 28

Topic for today
Periodic solutions

Reading for this lecture
BDH Section 5.5

Suggested exercises
Tutorial 8 questions

Reading for next lecture
BDH Section 3.6

Today's handouts
Lecture 27 notes

Example : Sketch the phase portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= -y \\ \frac{dy}{dt} &= 1 - 0.9y - x^2 - xy\end{aligned}$$

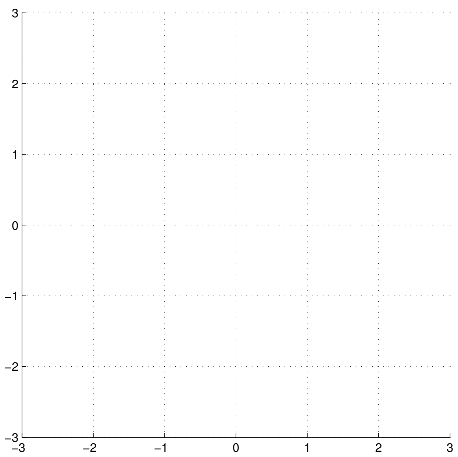
Equilibria

Jacobian

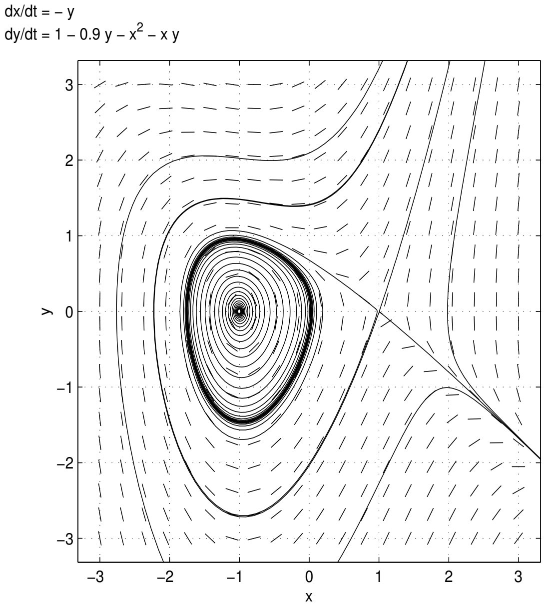
Solutions near $(1, 0)$

Solutions near $(-1, 0)$

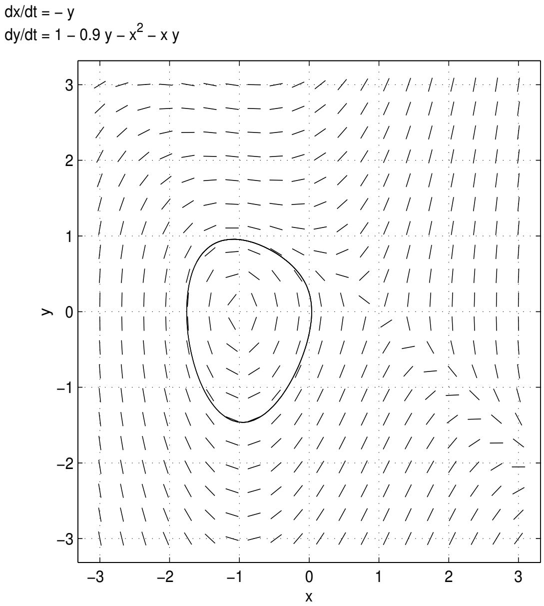
Nullclines



Phase portrait from *pplane*



Note how close together solution curves get in a band around the equilibrium at $(-1, 0)$. Careful use of *pplane* gives the following solution curve:



We see there is a closed solution curve passing close to the origin. This curve corresponds to a *periodic solution* of the system, i.e., a solution for which each dependent variable is a periodic function of time.

Example : Use *pplane* to investigate the qualitative changes in the behaviour of solutions to

$$
\begin{aligned}
\frac{dx}{dt} &= -y \\
\frac{dy}{dt} &= \lambda - 0.9y - x^2 - xy
\end{aligned}
$$

that occur as λ is varied in the interval $[-3, 3]$.

Some advanced features of *pplane* are useful for investigating this system. In particular, *pplane* can be used to do the following.

1. Plot nullclines. Select the 'Nullclines' option in the lower right of the 'Setup' window.
2. Find equilibria and determine their type. Select the option 'Find an equilibrium' from the 'Solutions' menu on the 'Display' window, move the cursor to a place in the display window near where you expect the equilibrium to be and click.
3. Plot solutions for t increasing only. In the 'Display' window, pull down the 'Options' menu, pick 'Solution direction' and then select 'Forward'.