Maths 260 Lecture 27

Topic for today
Sketching phase portraits for nonlinear systems

Reading for this lecture
BDH Section 5.2

Suggested exercises
Tutorial 8 questions

Reading for next lecture
BDH Section 5.5

Today's handout
Lecture 26 notes

Sketching phase portraits for nonlinear systems

We have learnt two methods for obtaining information about solutions to nonlinear systems:

1. Linearisation can give information about the behaviour of solutions near an equilibrium solution.

2. The method of nullclines gives information about where in the phase plane solution curves are horizontal and vertical. From this we can deduce where in the phase plane solutions move up, down, left or right.

We use both of these methods to sketch phase portraits for nonlinear systems.

Outline of method

To sketch a phase portrait, it can be helpful to follow some or all of the following steps.

1. Find all equilibria. Where possible, use linearisation to determine their types (e.g., saddle, spiral source).

2. Draw the nullclines. Determine the direction of the vector field in the regions between nullclines. Determine the direction of the vector field on the nullclines.

3. Sketch some representative solution curves. Make sure the solution curves you sketch go in the directions determined by the nullclines and behave like the appropriate linearised system near any equilibrium.

Note: Nullclines are **not** usually solution curves.

Example 1
Sketch the phase portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= 2 - x - y \\ \frac{dy}{dt} &= x^2 - y\end{aligned}$$

Determine the long term behaviour of the solutions through $(x, y) = (1, 2)$ and $(-3, 4)$.

Equilibria

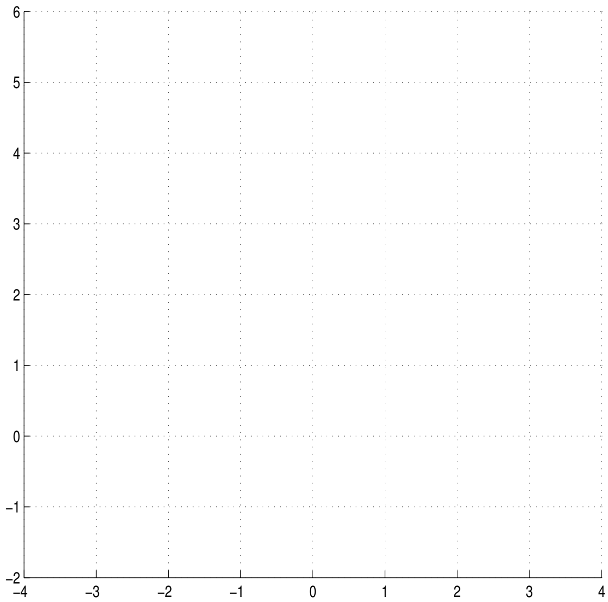
Jacobian

Solutions near $(1, 1)$

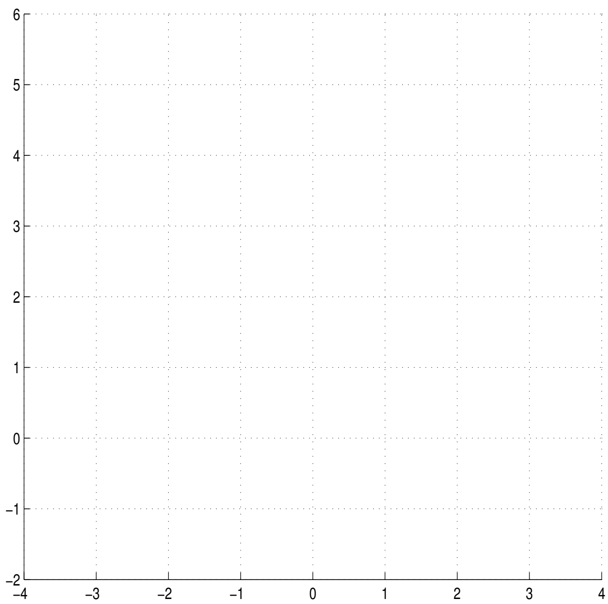
Solutions near $(-2, 4)$

Nullclines

Directions of solutions on and between nullclines



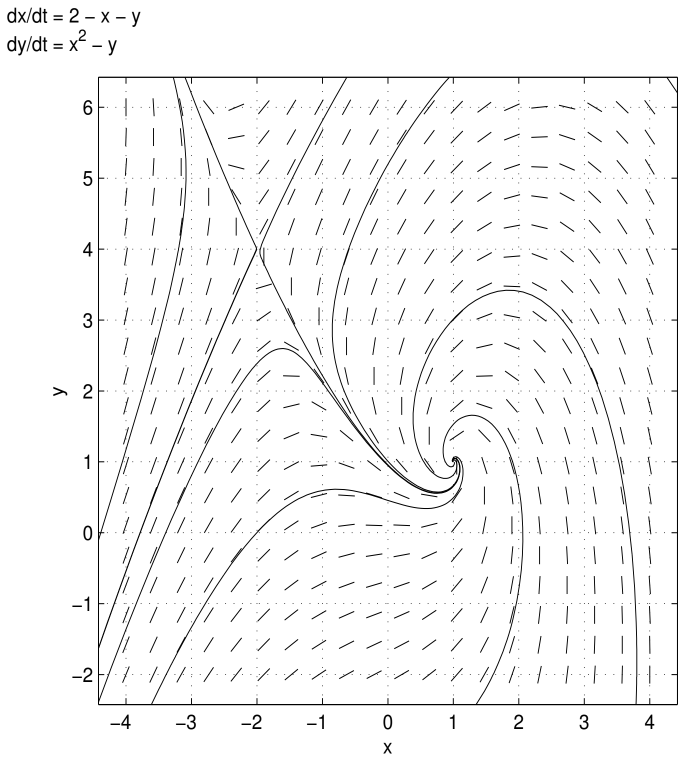
Sketch of phase portrait



Behaviour of solution through $(1, 2)$

Behaviour of solution through $(-3, 4)$

Phase portrait from *pplane*



Example 2

Sketch the phase portrait for the system

$$\begin{aligned}\frac{dx}{dt} &= x(x-1)\\ \frac{dy}{dt} &= x^2 - y\end{aligned}$$

Determine the long term behaviour of the solution through $(x, y) = (-1, 0)$, $(0.8, 0)$ and $(1, 3)$.

Equilibria

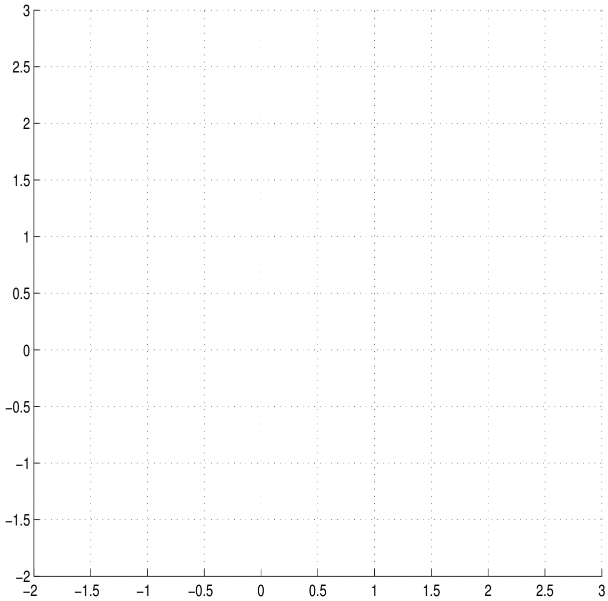
Jacobian

Solutions near $(0, 0)$

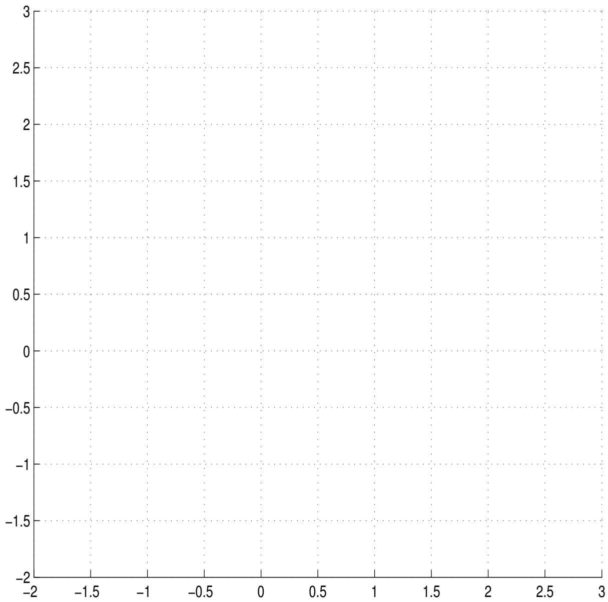
Solutions near $(1, 1)$

Nullclines

Directions of solutions on and between nullclines



Sketch of phase portrait



Behaviour of solution through $(-1, 0)$

Behaviour of solution through $(0.8, 0)$

Behaviour of solution through $(1, 3)$

Phase portrait from *pplane*