Maths 260 Lecture 1

Topics for today
Introduction to differential equations
Introduction to modelling

Reading for this lecture
BDH Section 1.1

Suggested Exercises
BDH Section 1.1: 1, 3, 13, 15

Reading for next lecture
BDH Section 1.2

Today's handouts
Course guide
Lecture 1 notes

Section 1.1 Modelling with Differential Equations

The subject of differential equations is about using derivatives to describe how a quantity changes.

Using knowledge about how a quantity changes to write down a DE is called *modelling*, and a DE is a *model*.

The goal of modelling is to use the DE model to predict future values of the quantity being modelled.

Today's class gives an overview of some types of models we look at in this course.

Important steps in making a model:

1. Identify assumptions on which the model is based.

2. Identify all relevant quantities in the model.

3. Use assumptions in (1) to write down equations relating the quantities in (2).

Quantities in a model divide into three types:

(a) independent variables

(b) dependent variables

(c) parameters

Keep the model as simple as possible!

Example 1: Single Population, Unlimited Growth

Assume: The population grows at a rate proportional to the size of the population.
Quantities:
t=time (independent variable)
P=size of population (dependent variable)
k=proportionality constant (parameter)
Model:

$$\frac{dP}{dt} \propto P \quad \text{or} \quad \frac{dP}{dt} = kP, \quad k > 0$$

Predictions of the model:

Example 2: Single Population, Limited Growth

Assume: If the population is small, the population grows at a rate proportional to the size of the population.
If the population is too large, the population will decrease.
Quantities:
t=time (independent variable)
P=size of population (dependent variable)
k=growth rate coefficient for small population
N=maximum size of population before growth negative
Model:

$$\frac{dP}{dt} = kP \times \text{something}$$

where 'something' ≈ 1 if P small and 'something' < 0 if $P > N$.

A simple model with these properties is:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right), \quad k > 0$$

Predictions of the model:

Example 3: Two Populations (Predator-Prey)

Quantities:
t=time (independent variable)
R=size of rabbit population (dependent variable)
H=size of hawk population (dependent variable)
a,b,c,f are various parameters
Model:

$$\begin{aligned} \frac{dR}{dt} &= aR\left(1 - R - \frac{bH}{1+2R}\right) \\ \frac{dH}{dt} &= cH\left(1 - \frac{fH}{R}\right) \end{aligned}$$

Important ideas/words from today
differential equation
ordinary differential equation
model
independent variable
dependent variable
parameter
first order differential equation
initial condition
qualitative analysis