Maths 260 Lecture 11

Topic for today
Bifurcations (continued)

Reading for this lecture
BDH Section 1.7

Suggested exercises
BDH Section 1.7: 11

Reading for next lecture
BDH Section 1.8

Today's handouts
Lecture 11 notes
Tutorial 4 question sheet

Section 1.7 continued: More on bifurcations

We are interested in one-parameter families of autonomous DEs:

$$\frac{dy}{dt} = f_\mu(y).$$

We look for bifurcations, i.e., changes in the qualitative behaviour of solutions as the parameter μ is varied.

General result about bifurcations

Bifurcations usually do not happen, i.e., a small change in the parameter usually leads to only a small change in the behaviour of solutions. To be precise, if

$$\frac{dy}{dt} = f_\mu(y)$$

where $\partial f/\partial \mu$ and $\partial f/\partial y$ exist and are continuous for all values of μ and y, then a small change in μ gives a small change in the graph of $f_\mu(y)$.

Example: Suppose the DE

$$\frac{dy}{dt} = f_\mu(y)$$

with $\mu = \mu_0$ has a source at $y = y_0$ with $df_{\mu_0}/dy > 0$. What is the effect on the qualitative behaviour of solutions of changing μ by a small amount?

A bifurcation where the number or type of equilibria changes can only occur at $\mu = \mu_0$ if

$$f_{\mu_0}(y_0) = 0 \text{ and } \frac{df_{\mu_0}}{dy}(y_0) = 0,$$

i.e., when the linearization theorem does not work.

Example: Draw the bifurcation diagram for the family of equations

$$\frac{dy}{dt} = \mu y + y^3$$

Example: Draw the bifurcation diagram for the family of equations

$$\frac{dy}{dt} = \mu y + y^2$$

Important ideas from today

Bifurcations are special: a small change in parameter does not usually result in a qualitative change in the behaviour of solutions.

A bifurcation where the number or type of equilibria changes can only occur at $\mu = \mu_0$ if

$$f_{\mu_0}(y_0) = 0 \text{ and } \frac{df_{\mu_0}}{dy}(y_0) = 0.$$