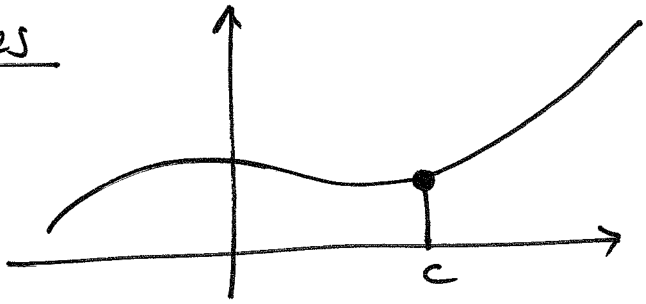
Continuous Functions

We require $f(c)$ to exist and

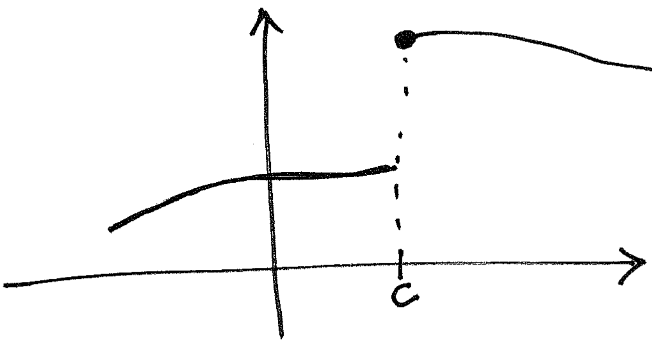
$$\lim_{x \to c} f(x) = f(c)$$

for f to be continuous at c.

Examples (1D)

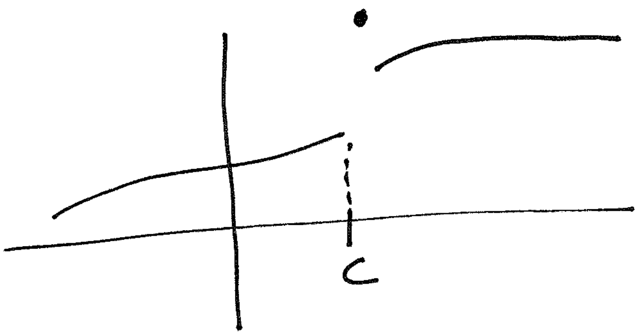


cts

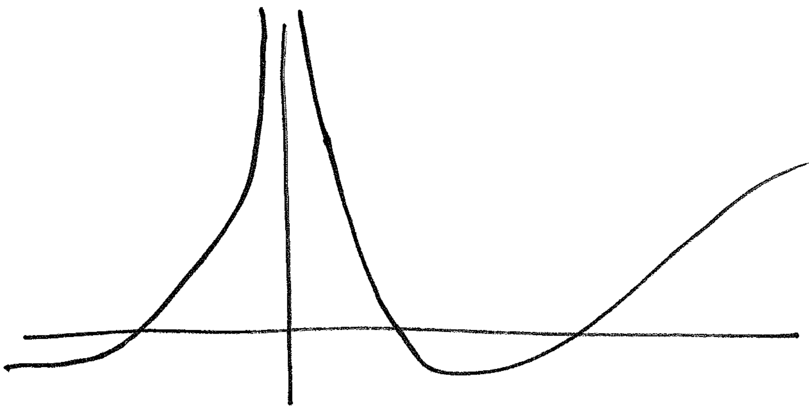


not cts

$\lim_{x \to c^-}$ & $\lim_{x \to c^+}$ must both be $f(c)$



not cts



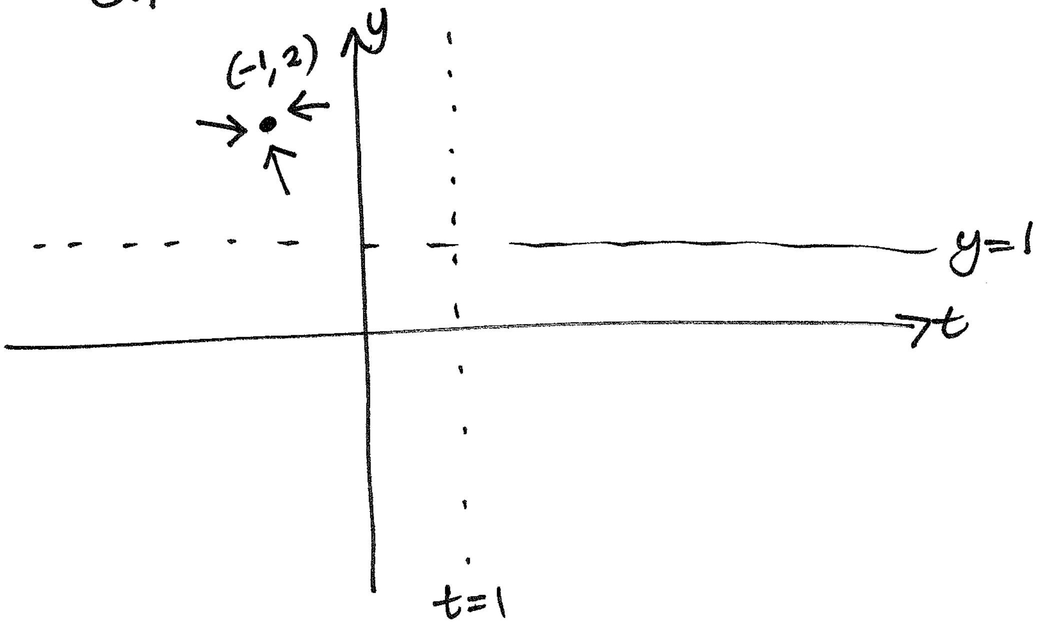
not cts at 0

$f(0)$ is undefined.

For more than 1-D, check the approach along various lines.

Example

$$\frac{dy}{dt} = (t-1)(y-1)^{1/3} = f(t,y)$$



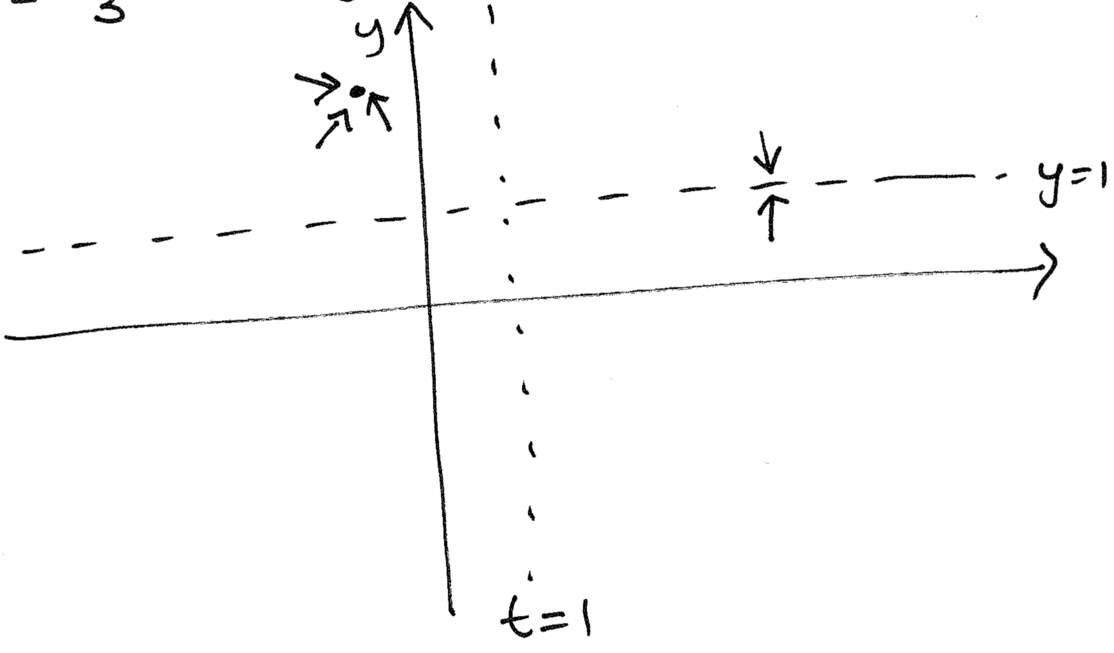
f is defined in whole of t,y plane & cts

eg $f(-1,2) = (-2)(1)^{1/3} = -2$

$$\lim_{\substack{t \to -1 \\ y \to 2}} f(t,y) = -2$$

cts.

$$\frac{\partial f}{\partial y} = \frac{1}{3}(t-1)(y-1)^{-2/3}$$ – not defined for $y=1$.



Polynomials are cts everywhere

Rational functions are cts where the denominator is non-zero.

If f and g are cts, then so is $f \circ g$.

Ex

$$h(x,y) = \cos(x^2+y^2)$$
$$= f(g(x,y))$$

where $g(x,y) = x^2+y^2$ cts

$f(\ ) = \cos(\ )$ cts

$\therefore$ h is cts.