Extra Notes on Harmonic Oscillator

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$b > \sqrt{4mk}$ (overdamping)

$$\lambda_1 = -\frac{b}{2m} + \frac{\sqrt{b^2-4mk}}{2m} < 0$$

$$\lambda_2 = -\frac{b}{2m} - \frac{\sqrt{b^2-4mk}}{2m}$$

Note that $\lambda_2 < \lambda_1 < 0$

The matrix describing the system is

$$A = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{pmatrix}$$

$$(A - \lambda I)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -\lambda & 1 \\ -\frac{k}{m} & -\frac{b}{m} - \lambda \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$= 0 \iff -\lambda v_1 + v_2 = 0$$

$$\iff v_2 = \lambda v_1$$

Eigenvectors are

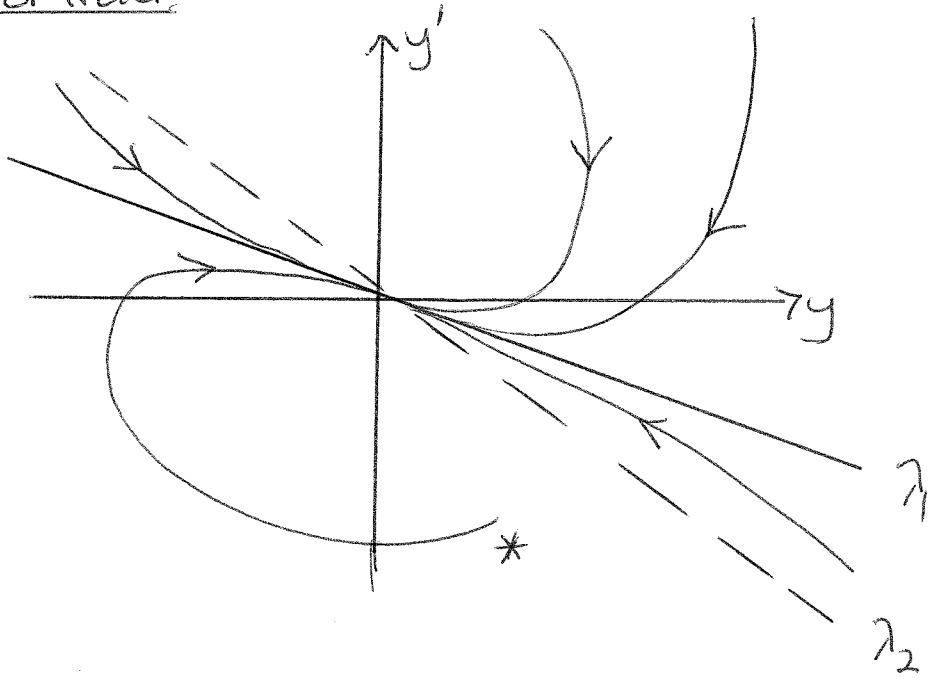
$$\lambda = \lambda_1, \quad \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix}$$

$$\lambda = \lambda_2, \quad \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix}$$

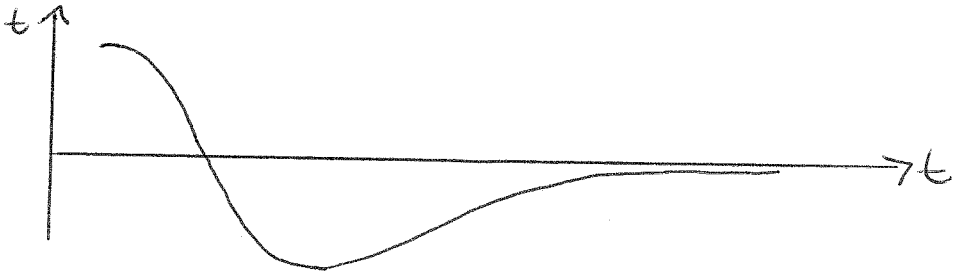
Since $\lambda_2 < \lambda_1 < 0$, the eigenvector with the least slope will be that corresponding to λ_1.

$b > \sqrt{4mk}$

Phase Portrait



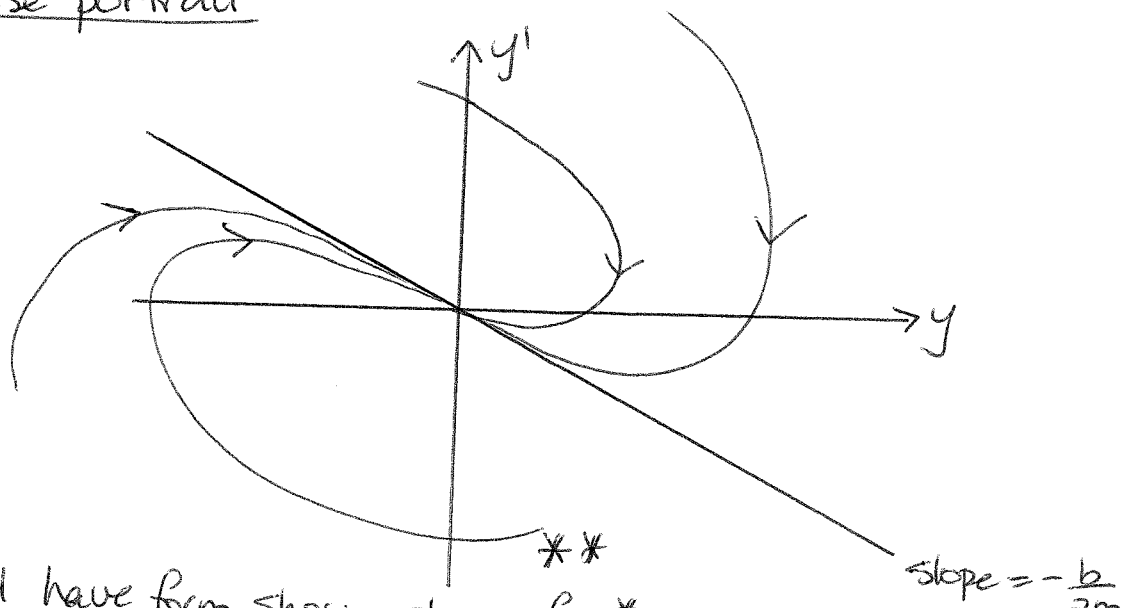
The soln * will be of the form below



$b = \sqrt{4mk}$ (Critical damping)

Imagine the two eigenvectors getting closer + closer. Get one eigenvector of slope $-b/2m$

Phase portrait



** will have form shown above for *