Department of Mathematics

Maths 260 Assignment 4

April 27, 2007 Due: 22 May 2007

Students should hand their assignments in at the Student Resource Centre in the basement of the Mathematics/Physics Building. Your completed assignment should be handed in to the appropriate box outside the Student Resource Centre **before** 4pm on the date due. Late assignments or assignments placed in the wrong box will not be accepted. Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box next to the Student Resource Centre.

1. (9 marks) Consider the following system of differential equations:

$$\frac{dY}{dt} = \begin{pmatrix} 0 & 2 \\ -2 & -1 \end{pmatrix}.$$

   (a) Find the eigenvalues and determine if the origin is a spiral source, spiral source or centre.

   (b) Find the direction of the oscillations (ie clockwise or anticlockwise).

   (c) Sketch the phase portrait.

   (d) Find the period of the oscillations.

   (e) Sketch the graph for $x(t)$ and for $y(t)$ if $x(0) = -1, y(0) = 1$.

2. (7 marks) Consider the following system of differential equations:

$$\frac{dY}{dt} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}.$$

   Find the general solution, carefully sketch the phase portrait and describe the long- term behaviour of the solutions.

3. (7 marks) Consider the following system of differential equations:

$$\frac{dY}{dt} = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}.$$

   (a) Find the general solution and carefully sketch the phase portrait.

   (b) Find the solution for which $Y(0) = (1, 3)^T$.

4. (12 marks) Consider the one parameter family of systems of differential equations:

$$\frac{dY}{dt} = \begin{pmatrix} 1 & 3 \\ -1 & a \end{pmatrix}.$$

   Determine the type of equilibrium (e.g. sink, source, saddle) at the origin for all values of a. Include any information about the number of straight line solutions.

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5. (15 marks) Consider the one parameter family of systems of differential equations:

$$\frac{dx}{dt} = y - x^2$$
$$\frac{dy}{dt} = y - a.$$

(a) Show that for $a < 0$, there are no equilibrium solutions.

(b) Find $a = 0$, find any equilibrium solutions.

(c) For $a = 1$:

i. Find all equilibrium solutions. What does linearisation tell you about the type of the equilibrium solutions?

ii. Find the nullclines for the system and sketch them using the grid below. Show the direction of the vector field in the regions between the nullclines and on the nullclines themselves.

iii. Sketch the phase portrait for the system. Include in the portrait the solutions curves through the initial conditions

A. $(x(0), y(0)) = (0, 2.5)$;

B. $(x(0), y(0)) = (0, 0.5)$.