Department of Mathematics

Maths 260 Assignment 3

April 2, 2007 Due: 8 May 2007

Students should hand their assignments in at the Student Resource Centre in the basement of the Mathematics/Physics Building. Your completed assignment should be handed in to the appropriate box outside the Student Resource Centre **before** 4pm on the date due. Late assignments or assignments placed in the wrong box will not be accepted. Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box next to the Student Resource Centre.

1. (10 marks) In Lecture 1, we saw the following predator-prey model for rabbits and hawks:

$$\frac{dR}{dt} = 0.1R(1 - R - \frac{0.2H}{1+2R})$$
$$\frac{dH}{dt} = 0.2H(1 - \frac{0.1H}{R}).$$

   Assume the populations are measured in thousands.

   (a) Find all equilibrium solutions (that make sense for a population model!).

   (b) Use `pplane` to show the phase plane for the system. On the phase plane show the equilibrium solutions you found in (a).

   (c) Sketch a graph of R against t, and of H against t when the initial populations are:

      i. 2000 rabbits and 1000 hawks

      ii. 400 rabbits and 8000 hawks

2. (6 marks) Find the general solution of

$$\frac{dx}{dt} = -\frac{x}{t} + y$$
$$\frac{dy}{dt} = 2ty.$$

3. (10 marks) Find the general solution of

$$\frac{dx}{dt} = x - 2y$$
$$\frac{dy}{dt} = 3x - 4y.$$

   Sketch the phase portrait for the system.

4. (9 marks) Find the general solution of

$$\frac{dY}{dt} = \begin{pmatrix} 1 & -0 & 0 \\ 0 & 3 & -2 \\ 0 & 4 & -1 \end{pmatrix} Y.$$