# Maths 260 Assignment 2

March 19, 2007 Due: 4pm, Tuesday, 3 April 2007

Students should hand their assignments in at the Student Resource Centre in the basement of the Mathematics/Physics Building. Your completed assignment should be handed in to the appropriate box outside the Student Resource Centre **before** 4pm on the date due. Late assignments or assignments placed in the wrong box will not be accepted. Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box next to the Student Resource Centre.

**1.** Consider the the initial value problem

$$\frac{dy}{dt} = 1 - t + 4y, \qquad y(0) = 1.$$

By hand, estimate $y(0.2)$ using:

(a) 2 steps of Improved Euler method

(b) 1 step of 4th order Runge-Kutta method.

Show your working. You may use MATLAB to check your answers but not to get them.

**2.** Find the analytical solution for the initial value problem in Question 1 and use it to find the errors in your two estimates of $y(0.2)$. Comment on the errors.

**3.** The following table shows the results of using a numerical method to estimate the solution at $t = 1$ of IVP

$$\frac{dy}{dt} = y, \qquad y(0) = 1.$$

| No. steps | Est. of y(1) | Error |
|---|---|---|
| 1 | 2.6666667 | 5.1615162e-02 |
| 2 | 2.7087674 | 9.5144673e-03 |
| 4 | 2.7168320 | 1.4498551e-03 |
| 8 | 2.7180816 | 2.0019857e-04 |
| 16 | 2.7182555 | 2.6304454e-05 |
| 32 | 2.7182785 | 3.3711754e-06 |
| 64 | 2.7182814 | 4.2669349e-07 |

(a) What is the effective order at

i. $h = 0.125$?

ii. $h = 0.03125$?

(b) Estimate the order of the method. Give reasons for your answer.

**4.** Consider the differential equation

$$\frac{dy}{dt} = 3(t+y)^{1/2}.$$

(a) Use `dfield` to show the direction field. Hand in a printout.

(b) Use the existence theorem to find out for what initial conditions $(t_0, y_0)$ a solution exists. Does this agree with what you observe on the direction field?

(c) Use the existence and uniqueness theorem to find out for what initial conditions $(t_0, y_0)$ a unique solution exists. Does this agree with what you observe on the direction field?

**5.** Consider the differential equation

$$\frac{dy}{dt} = y^2 - y^3.$$

(a) Find all the equilibrium solutions and sketch the phase line.

(b) Classify all the equilibrium solutions.

(c) Use the phase line to sketch solutions for the following initial conditions:

i. $y(0) = 2$

ii. $y(0) = 0.5$

iii. $y(0) = -0.5$

**6.** A small population of a species of fish is growing at the rate of 10% per year, living in a lake which can sustain a maximum population of 100,000 of this species. It has been decided to allow fishing of the species so the population has to be modelled with harvesting. Let $\alpha$ be number that can be fished per year (measured in thousands).

- Write a differential equation to model the population.
- If $\alpha = 1.5$, what is the minimum number of fish required in the population before fishing starts? Give reasons for your answer.
- Draw a bifurcation diagram for the differential equation.

**7.** Show that if $a$ and $\lambda$ are positive constants, where $a \neq \lambda$, and $b$ is any real number, then for every solution of the differential equation

$$\frac{dy}{dt} + ay = be^{-\lambda t}$$

$y \to 0$ as $t \to \infty$.

Solve the differential equation for $a = \lambda$, and plot the solution using `analyzer`. What do you think the $\lim_{t\to\infty} y$ is? Hand in plots that illustrate your answer.