DEPARTMENT OF MATHEMATICS

# Maths 260 Assignment 1 Sample Solutions

March 20, 2007 Due: 4pm, Tuesday, 13 March 2007

**1.** (a) i. $\frac{dy}{dt} = 3$ and $\frac{2(y-1)}{t} - 3 = \frac{2(3t)}{t} - 3 = 3$. Therefore $y_1$ is a solution.

ii. $\frac{dy}{dt} = 4t + 3$ and $\frac{2(y-1)}{t} - 3 = \frac{2(2t^2+3t)}{t} - 3 = 4t + 3$. Therefore $y_2$ is a solution.

iii. $\frac{dy}{dt} = 4t+4$ and $\frac{2(y-1)}{t} - 3 = \frac{2(2t^2+4t-3)}{t} - 3 = 4t - \frac{6}{t} + 5$. Therefore $y_3$ is not a solution.

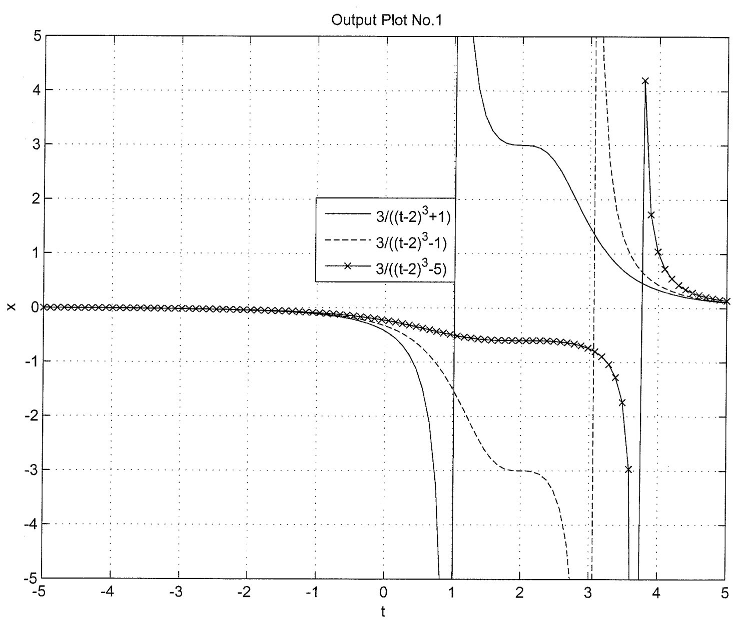
(b) Only $y_1$ satisfies $y(1) = 4$. So $y_1$ is a solution of the IVP.

**2.** (a)

$$\begin{aligned} -\int \frac{dy}{y^2} &= \int (t-2)^2 dt \\ \frac{1}{y} &= \frac{1}{3}(t-2)^3 + c \\ y &= \frac{3}{(t-2)^3 + 3c} \\ &= \frac{3}{(t-2)^3 + k} \end{aligned}$$

(b) $y = 0$ is also a solution. There is no choice of $k$ that gives this solution.

(c) Analyzer produces




Note that the plots have vertical asymptotes.

(d) $y(0) = \frac{3}{-8+k} = 3$ which requires that $k = 9$. Therefore $y = \frac{3}{(t-2)^3+9}$.

**3.** (a)

$$\frac{dT}{dt} = k(T - T_a),$$

where $T(t)$ is the temperature at time $t$ and $T_a$ is the ambient temperature.

(b) That the body is moved instantaneously to the morgue. That the ambient temperature where the body was found has been the same from the time the person died until the body was found. That the person had normal body temperature when they died.

(c)

$$\begin{aligned}
\frac{dT}{dt} &= k(T-5) \text{ while the body is in the morgue} \\
\int \frac{dT}{T-5} &= \int k dt \\
\ln|T-5| &= kt + c \text{Note that } T > 5 \\
T - 5 &= e^{kt+c} \\
&= Ke^{kt} \\
T &= 5 + Ke^{kt}
\end{aligned}$$

Since $T(0) = 30$, $5 + K = 30$ and so $K = 25$.

$$T(t) = 5 + 25e^{kt}.$$

The temperature of the body after one hour in the morgue is known to be 27, therefore $T(1) = 27 = 5 + 25e^k$ so $e^k = 22/25$ and $k = \ln 22/25 \approx -0.1278$.

$$T(t) = 5 + 25e^{-0.1278t}.$$

(d) For the model before the body was found, use

$$\frac{dT}{dt} = k(T - 15), \qquad T(0) = 30.$$

The solution is

$$T(t) = 15 + 15e^{-0.1278t}.$$

The person died when the temperature of their body was 37.

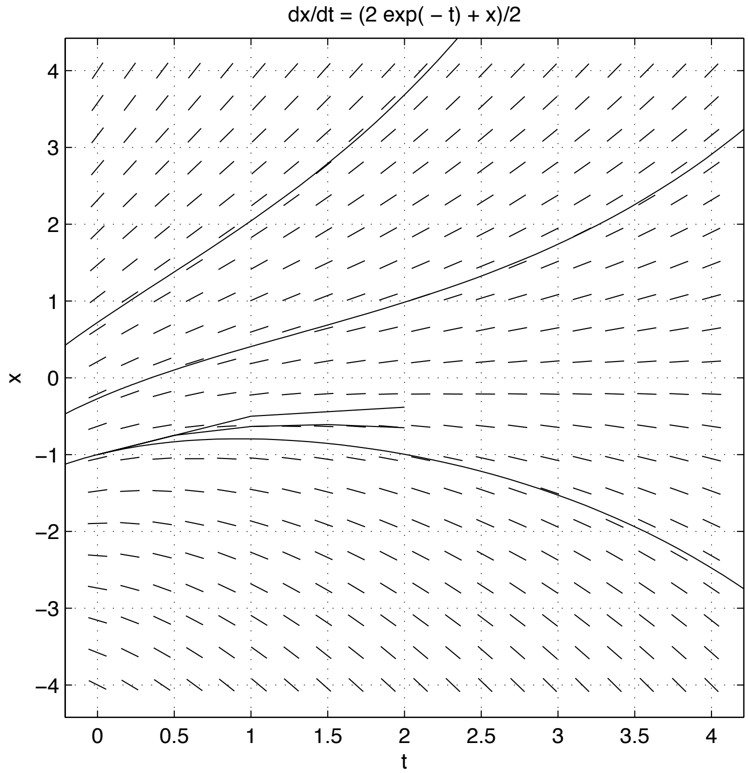
$$\begin{aligned}
37 &= 15 + 15e^{-0.1278t} \\
22 &= 15e^{-0.1278t} \\
e^{-0.1278t} &= \frac{22}{15} \\
t &= -\frac{1}{0.1278} \ln \frac{22}{15} \\
&= -2.99.
\end{aligned}$$

Therefore the person died approximately 3 hours before being found, i.e. at about 9pm.

**4.** (a) See below.

(b) See below.

(c) Parts (a), (b) and (c) give




(d) The error for $h = 1$ is about 0.6 and for $h = 0.5$ it is about 0.35. The one for $h = 0.5$ is better because the error is smaller. It is what is expected because normally error is divided by 2 when the stepsize is divided by 2.

(e) i. Euler's method with $h = 0.01$, needing 2 steps.

| $n$ | $t_n$ | $y_n$ | $f(t_n, y_n)$ |
|---|---|---|---|
| 0 | 0 | 1 | 0.5 |
| 1 | 0.01 | $-1 + 0.01 \times 0.5 = -0.995$ | 0.492550 |
| 2 | 0.02 | $-0.995 + 0.01 \times 0.492550 = -0.990075$ | |

ii. Euler's method with $h = 0.005$, needing 4 steps.

| $n$ | $t_n$ | $y_n$ | $f(t_n, y_n)$ |
|---|---|---|---|
| 0 | 0 | $-1$ | 0.5 |
| 1 | 0.005 | $-0.9975$ | 0.496262 |
| 2 | 0.01 | $-0.995019$ | 0.492540 |
| 3 | 0.015 | $-0.992556$ | 0.488834 |
| 4 | 0.02 | $-0.990112$ | |