Department of Mathematics

Maths 260 Assignment 1

March 8, 2007 Due: 4pm, Tuesday, 13 March 2007

Students should hand their assignments in at the Student Resource Centre in the basement of the Mathematics/Physics Building. Your completed assignment should be handed in to the appropriate box outside the Student Resource Centre **before** 4pm on the date due. Late assignments or assignments placed in the wrong box will not be accepted. Your assignment **must** be accompanied by a blue Mathematics Department coversheet. Copies of the coversheet are available from a box next to the Student Resource Centre.

1. (a) Determine whether any of the following functions is a solution to the differential equation

$$\frac{dy}{dt} = \frac{2(y-1)}{t} - 3.$$

i. $y_1(t) = 3t + 1$
ii. $y_2(t) = 2t^2 + 3t + 1$
iii. $y_3(t) = 2t^2 + 4t - 2$

(b) Are any of the functions in (a) a solution to the initial value problem consisting of the above differential equation with the initial condition $y(1) = 4$? Give reasons for your answers.

2. Consider the differential equation

$$\frac{dy}{dt} = -y^2(t-2)^2.$$

(a) Use separation of variables to solve the differential equation.

(b) Are there any solutions that are missing from the set of solutions that you found in 2a?

(c) Use *analyzer* to draw three different solutions to the differential equation. Draw all three graphs on the same picture, printout your picture, and hand it in with your assignment.

(d) Solve the initial value problem consisting of the differential equation and the initial condition $y(0) = 3$.

3. The police find a body and have it taken to the morgue at midnight when its temperature is 30°C. Where the body is found the ambient temperature is 15°C, and at the morgue it is 5°C. After one hour in the morgue the body's temperature is 27°C. The normal body temperature is 37°C.

Newton's law of cooling states: *The rate of change of temperature of an object is proportional to the difference between the temperature of the object and the ambient temperature.*

(a) Write Newton's law of cooling as a differential equation, using parameters for the ambient temperature and the constant of proportionality, k.

(b) What assumptions will you need to make so that this differential equation can be used to find a function describing the temperature of the body?

(c) Use the information given to find the constant of proportionality for the body as follows:

- Let $t = 0$ be the time when the body is found, so $T(0) = 30$.
- Solve the IVP with your differential equation from (a), and initial condition $T(0) = 30$.
- Your solution will have k in it. From that solution, write down $T(1)$ in terms of k.
- Put $T(1) = 27$ and solve for k.

(d) Write down the expression for $T(t)$ from (c). Use the information given to find the time of death of the body.

4. Consider the initial value problem

$$\frac{dy}{dt} = \frac{2e^{-t} + y}{2}$$

for $t \in [0, 4]$, with initial condition $y(0) = -1$.

(a) Use the tool *dfield* from *Matlab* to draw the slope field for the differential equation. Print it.

(b) On the printout from (a), sketch by hand, as accurately as you can, three approximate solutions to the differential equation, including the solution that satisfies the initial condition given above.

(c) On the same printout, draw by hand (and ruler) the graph you would obtain if you used Euler's method, with stepsize $h = 1.0$, to approximate the solution to the initial value problem at $t = 2.0$. You do not need to perform any calculations with Euler's method to answer this part of the question - just use the slope marks shown on your printout. Repeat for $h = 0.5$.

(d) Estimate the error in each of your Euler approximations, using the solution you sketched in (b). Which of your approximations is more accurate? Is this what you expect? Give a reason for your answer.

(e) Use Euler's method, with stepsize (i) h=0.01 and (ii) h=0.005, to calculate an approximation to the solution of the initial value problem at $t = 0.02$. Retain six significant figures of accuracy in your answer and show a table of n, t_n, y_n and $f(t_n, y_n)$. Show your working.